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Method for optimizing multi-layer insulation

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METHOD FOR OPTIMIZING

MULTI-LAYER INSULATION

BY 541

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ABSTRACT

The use of multi-layer insulation can result in reduced material cost, weight, or volume. A method for selecting layer materials is described. Each layer material is selected to operate at its upper temperature limit and to contribute to the overall performance of the wall. The analyses required for optimizing flat and cylindrical walls are developed.

Use of the digital computer makes it feasable to select layer materials from a large group of candidates. Fortran IV programs for optimizing flat and cylindrical walls are presented, and their use is demonstrated by the solution of example problems.

Methods for improving the usefulness of these programs are outlined.

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NOMENCLATURE

ALGEBRAIC

A - Surface Area, ft^2

B - Proportionality Constant, inch-BTU/ft^2-hr

C - Constant, \$/ft^3, lb/ft^3, or ft^3/ft^3

D - Proportionality Constant, \$-BTU/ft^4-hr, lb-BTU/ft^4-hr, or ft^3-BTU/ft^4-hr

h - Convection Heat Transfer Coefficient, BTU/hr-ft^2-°F

k - Thermal Conductivity, BTU/hr-ft-°F

ℓ - Length, ft

N - Maximum Number of Wall Layers

P - Cost, dollars; Weight, lb; or Volume, ft^3

q - Heat Transfer Rate, BTU/hr

R - Radius, ft

S - Cost per Unit Volume, \$/ft^3

t - Temperature, °F

Δt - Temperature drop, °F

V - Volume, ft^3

X - Thickness, inches

ρ - Mass Density, lb/ft^3

SUBSCRIPTS

A - Allowable heat loss

C - Cold Surface or fluid

h - Hot Surface or convective value

i - Any Location from 1 to n in Material Array

j - Any Wall Layer from 1 to n

j_n - Iteration Values of Layer Radius, n=1,2,3, etc.

min - Minimum

n - Last Layer of a Wall or Last Material of Array

o - Refers to Radius of Cylinder to be Insulated

FORTRAN

A - Surface Area, ft^2

AK - Conductivity, BTU/hr-ft-°F

CON - Conductivity, BTU/hr-ft-°F

DT - Temperature Drop, °F

H - Convection Heat Transfer Coefficient, BTU/hr-ft^2-°F

I - Material Array Index

J - Wall Layer Index

L - Length of Cylinder, ft

NINP - Output Option Control Input

NL - Layer Limit Control Input

NPROG - Optimization Option Control Input

Q - Heat Flow Rate, BTU/hr

RBIG - Large Radius of a Layer, ft

RSML - Small Radius of a Layer, ft

RO - Density, lb/ft^3

RIN - Radius of Cylinder to be Insulated, ft

SU - Cost per Unit Volume, \$/ft^3

TC - Cold Temperature of Wall or Fluid, °F

TH - Hot Temperature of Wall, °F

T - Allowable Operating temperature of a Material or Surface temperature of a layer, °F

INTRODUCTION

Thermal insulation is employed for a variety of reasons. It is used to provide a safe or compatible environment for men, equipment, and materials. It is also used to reduce energy loss, and it is used to maintain desired process temperatures. Regardless of the reason for using thermal insulation, the desired objectives are accomplished because heat flow and/or temperature are controlled.

Many insulation problems can be stated in terms of steady, one dimensional heat flow, temperature, and geometry requirements. The proper thickness of a material that will withstand the operating temperature range will be a solution to a particular problem (assuming the material is compatible with other environmental conditions); however, an optimum solution which results in minimum cost, weight, or volume is usually desired. Cost is of major importance to process, power generation, and fabrication industries where large quantities of insulation materials are involved. Minimum weight or volume are often of primary concern to the aerospace and transportation industries.

It may be necessary to use more than one layer of insulation to obtain an optimum solution to a problem. For industrial applications, a multi-layer wall should be considered when the single layer solution has a thickness of three or more inches (1)*. In addition, many high temperature applications require the use of composite walls to minimize thermal gradient stressing of the insulation (2).

---
* Denotes References

Selection of the optimum insulation configuration requires knowledge of all possible candidate materials and a comparison of the analyses of all possible composite walls constructed of these materials. The first, or hottest layer, of the composite wall is composed of the material which would result in the optimum single layer wall. It is selected from all materials with maximum operating temperatures equal to or above the hot temperature of the problem statement. The second layer is composed of the material which would be used with the optimum two layer wall. It would be selected from all materials with maximum operating temperatures less than the hot temperature of the problem statement and greater than the cold temperature of the problem statement. The third and subsequent layers are selected by the same method as the second.

Because of the large number of available insulation materials, the manual computation of an optimum composite wall can become extremely time consuming. However, the repetitive nature of these calculations is ideally suited to digital computer solution. Solution techniques for flat and cylindrical composite wall configurations were programmed for the IBM System 360/50 computer in the Fortran IV language. The program listings are presented in Appendices I and II. These computer programs may be used to solve directly for the composite wall of minimum cost, weight, or volume. The programs can also be used in the solution of the type of problem where it is desired to obtain a composite wall configuration that results in overall economy when variables such as cost of heat energy, installation cost, and cost of capital at various periods of amortization are to be considered.

In such cases the minimum cost wall may be determined as a function of heat flow rate for different numbers of layers and the results cross plotted with the other variables as a function of heat flow rate.

METHOD OF APPROACH

The methods for determining optimum flat and cylindrical composite insulation configurations are similar. In each case a minimum cost, weight or volume configuration is defined by the proper selection and sizing of layer materials.

The simplifying assumptions and basic materials selection procedures are described in this section. Detailed methods of analyses are presented in the DETAILED ANALYSIS SECTION.

A. ASSUMPTIONS

Unless otherwise stated the following assumptions are applicable to both flat and cylindrical insulation configurations:

1. Material Selection Criteria

It is assumed that an insulating material may be selected on the basis of two criteria. The first criterion is that the service temperature of the material is not exceeded by the maximum temperature it will encounter in the application. The second criterion is that the material of any additional layer provide improved performance over the previous layer.

2. Mode of Heat Transfer

It is assumed that heat transfer in the composite wall is by conduction alone. The thermal contact resistances between adjacent layers are neglected for simplicity. To account for the variation of surface area and temperature with radius, convection at the exterior surface of the cylindrical wall is considered. A constant value of film coefficient is assumed. For simplicity the heat transfer external to the flat wall and at the interior of the cylindrical wall is not considered.

3. Restriction on Conduction Equation

Conduction is assumed to be steady with time and one-dimensional in the direction perpendicular to the surface of the wall. The material of each layer is homogeneous in the direction of heat flow, of constant conductivity, and free of sources and sinks.

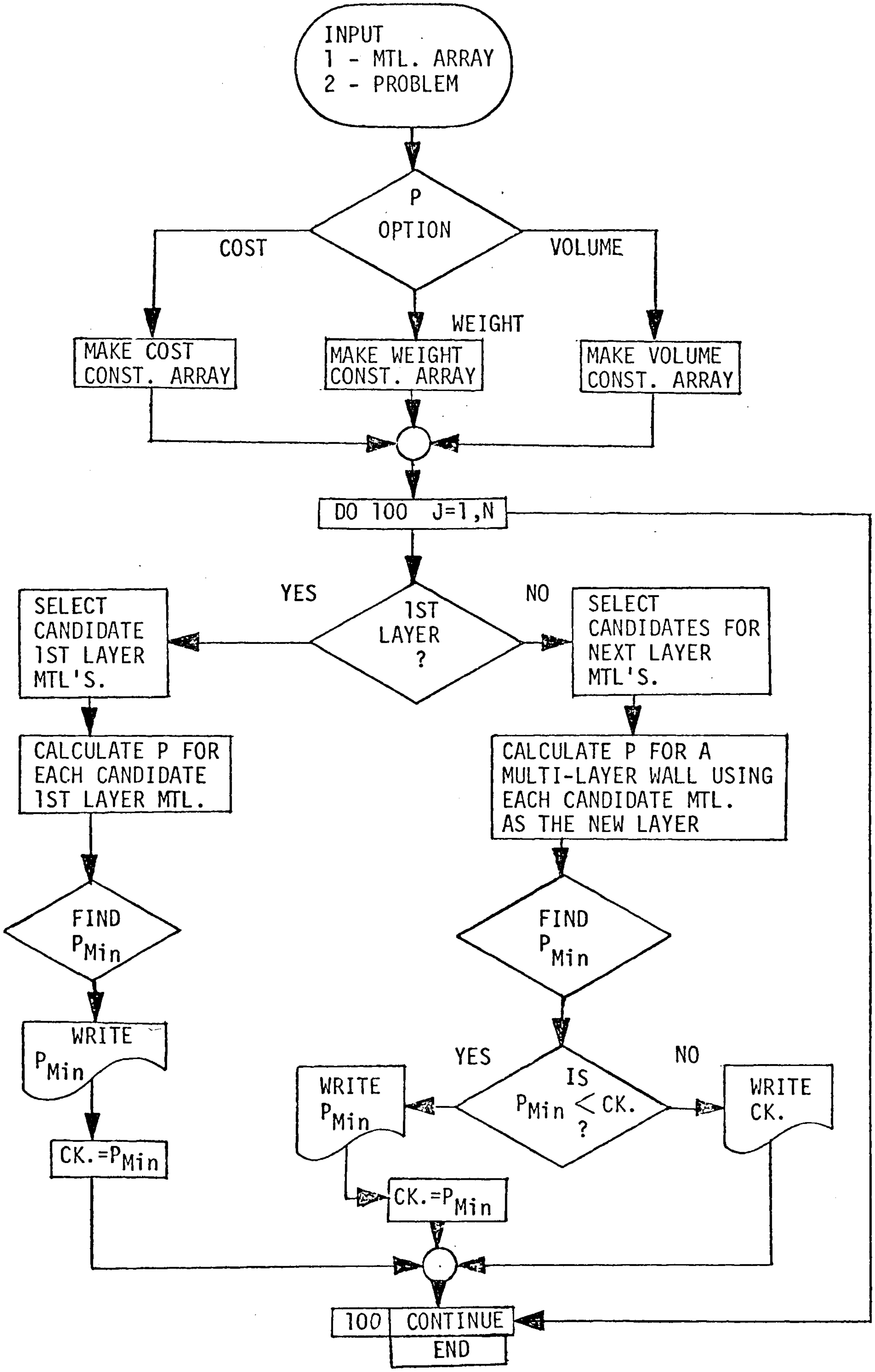
B. BASIC SELECTION PROCEDURE

The procedures for selecting optimum flat and cylindrical composite walls follow the same general logic pattern. The diagram of this flow pattern is shown in Fig. 1.

Input consists of an array of material data, a problem statement, and necessary control numbers. The maximum operating temperature, conductivity, cost per unit volume, and density of each material are input as follows:

$$
\begin{array}{l}
t_1, k_1, S_1, \rho_1 \\
t_2, k_2, S_2, \rho_2 \\
\vdots \\
t_i, k_i, S_i, \rho_i \\
\vdots \\
t_n, k_n, S_n, \rho_n \; .
\end{array}
$$

The flat wall problem statement lists the hot and cold surface temperatures, heat flow rate, and surface area as t_h, t_c, q, and A, respectively. The cylindrical problem statement lists the internal hot surface temperature of the cylinder, fluid temperature, heat flow rate, length of cylinder, film coefficient, and internal radius as t_h, t_c, q, ℓ, h, and R_0 respectively.


INPUT
1 - MTL. ARRAY
2 - PROBLEM
P OPTION
COST
WEIGHT
VOLUME
MAKE COST CONST. ARRAY
MAKE WEIGHT CONST. ARRAY
MAKE VOLUME CONST. ARRAY
DO 100 J=1,N
YES
NO
1ST LAYER ?
SELECT CANDIDATE 1ST LAYER MTL'S.
SELECT CANDIDATES FOR NEXT LAYER MTL'S.
CALCULATE P FOR EACH CANDIDATE 1ST LAYER MTL.
CALCULATE P FOR A MULTI-LAYER WALL USING EACH CANDIDATE MTL. AS THE NEW LAYER
FIND P_{Min}
FIND P_{Min}
WRITE P_{Min}
$CK.=P_{Min}$
YES
NO
IS $P_{Min} < CK.$?
WRITE P_{Min}
WRITE CK.
$CK.=P_{Min}$
100 CONTINUE
END

Following the option selection, a list of material constants, c_i, equal to S_i, ρ_i, or unity is stored respectively for the minimum cost, weight, or volume option.

Next a control loop is entered. The control loop requires the operations that follow it to be executed N (an input control number) times. On the first pass through the control loop, the optimum first layer material is selected and optimum single layer wall is sized by performing the operations on the left side of Fig. 1. A new layer material is selected on each subsequent pass through the loop. If the new optimum wall has a lower value of P than the previous optimum wall its description is written.

First layer candidate materials have allowable operating temperatures that are equal to or greater than the hot temperature of the problem statement, i.e., $t_i \geq t_h$. The parameter P is the cost, weight, or volume of the wall that satisfies the problem statement. For a single layer flat wall,

$$P = \frac{A^2}{q}\left(C_i k_i t_h - C_i k_i t_c\right). \tag{1}$$

For a single layer cylindrical wall,

$$P = \pi \ell \left(C_i R_1^2 - C_i R_o^2\right) . \tag{2}$$

P values are calculated for each first layer candidate material, and the material that produces the lowest value of P is selected as the optimum first layer material.

On each subsequent pass through the loop candidate, next layer materials are selected. These candidate materials have operating temperature limits which are between the hot and cold temperatures given in the problem statement, i.e., $t_\ell > t_i > t_c$. The multi-layer flat wall P equation is

$$P = \frac{A^2}{q}\left(k_1 C_1 t_\ell - k_1 C_1 t_2 + k_2 C_2 t_2 - k_2 C_2 t_3 \cdots \right.$$

$$+ k_j C_j t_j - k_j C_j t_{(j+1)} \cdots$$

$$\left. + k_n C_n t_n - k_n C_n t_c \right) . \tag{3}$$

For the cylindrical wall

$$P = \pi \ell \left[- C_1 R_0^2 + C_1 R_1^2 - C_2 R_1^2 + C_2 R_2^2 \cdots \right.$$

$$\left. - C_j R_{(j-1)}^2 + C_j R_j^2 \cdots - C_n R_{(n-1)}^2 + C_n R_n^2 \right] . \tag{4}$$

The thickness of the previous layer is reduced when a new layer is added. This thickness change results from changing the lower surface temperature from the cold temperature of the problem statement to the maximum operating temperature of the new layer material.

DETAILED ANALYSIS

In this section parameter equations are developed for the flat and cylindrical composite wall configurations. Techniques required to determine the internal and external radii of a cylindrical insulation layer are also presented.

A. FLAT WALL

Equations for calculating the volumes of each of the layers of an n layer composite wall are as follows:

$$
\begin{aligned}
V_1 &= A\,X_1 \\
V_2 &= A\,X_2 \\
&\vdots \\
V_j &= A\,X_j \\
&\vdots \\
V_n &= A\,X_n\,.
\end{aligned}
$$

In these equations A is the surface area, and each X_j is a layer thickness. By multiplying each volume equation by a constant, c_j, the set of parameter equations

$$
\begin{aligned}
P_1 &= c_1 V_1 = A\,c_1 X_1 \\
P_2 &= c_2 V_2 = A\,c_2 X_2 \\
&\vdots \\
P_j &= c_j V_j = A\,c_j X_j \\
&\vdots \\
P_n &= c_n V_n = A\,c_n X_n
\end{aligned}
\tag{5}
$$

are obtained in which each P_j parameter represent the cost, weight, or volume of a layer where each C_j value is equal respectively to the layer cost per unit volume, or density, or unity.

From Fourier's law, the thicknesses of wall layers are as follows:

$$
\begin{aligned}
X_1 &= \frac{A}{q} k_1 \Delta t_1 = \frac{A}{q} k_1 (t_h - t_2) \\
X_2 &= \frac{A}{q} k_2 \Delta t_2 = \frac{A}{q} k_2 (t_2 - t_3) \\
&\vdots \\
X_j &= \frac{A}{q} k_j \Delta t_j = \frac{A}{q} k_j (t_j - t_{(j+1)}) \\
&\vdots \\
X_n &= \frac{A}{q} k_n \Delta t_n = \frac{A}{q} k_n (t_n - t_c) \ . \qquad (6)
\end{aligned}
$$

Each k_j value is the conductivity of the respective j layer material and q is the heat flow rate that passes through each layer of the composite wall. Each pair of t_j and $t_{(j+1)}$ values are respectively the hot and cold surface temperatures of the X_j thick layer. The hot surface temperature of the first layer is the hot surface temperature of the wall (t_h) given in the problem statement; and likewise, the cold temperature of the last layer is the cold temperature (t_c) of the problem statement.

By substituting equations (6) into equations (5), the set of layer parameter equations are obtained as follows:

$$P_1 = \frac{A^2}{q} k_1 C_1 (t_k - t_2)$$

$$P_2 = \frac{A^2}{q} k_2 C_2 (t_2 - t_3)$$

$$\vdots$$

$$P_j = \frac{A^2}{q} k_j C_j (t_j - t_{(j+1)})$$

$$\vdots$$

$$P_n = \frac{A^2}{q} k_n C_n (t_n - t_c) \ . \tag{7}$$

The parameter equation for the total composite wall is obtained by summation of the above P_j equations. The composite wall equation is

$$\begin{aligned} P = \frac{A^2}{q} (&k_1 C_1 t_k - k_1 C_1 t_2 + k_2 C_2 t_2 - k_2 C_2 t_3 \cdots \\ &+ k_j C_j t_j - k_j C_j t_{(j+1)} \cdots \\ &+ k_n C_n t_n - k_n C_n t_c) \ . \end{aligned}$$

This equation was originally presented as equation (3) in the last section. The single layer equation, equation (1), is the single layer application of equation (3).

B. CYLINDRICAL WALL

1. Derivation of Parameter Equation

Let R_0 and ℓ be the radius and length of a cylinder which is to be insulated. If $R_1, R_2, \cdots R_j \cdots R_n$ are the outer radii of the insulation layers applied to the cylinder, then the volumes of each of the layers are as follows:

$$V_1 = \pi \ell \left[R_1^2 - R_0^2 \right]$$
$$V_2 = \pi \ell \left[R_2^2 - R_1^2 \right]$$
$$\vdots$$
$$V_j = \pi \ell \left[R_j^2 - R_{(j-1)}^2 \right]$$
$$\vdots$$
$$V_n = \pi \ell \left[R_n^2 - R_{(n-1)}^2 \right] \quad .$$

Let the parameter P_j be the total material cost, weight, or volume of any j layer of material, and let C_j respectively represent the per unit volume, cost, weight, or volume of the layer material. The P_j values (cost, weight, or volume) of each layer of an n layer composite wall are represented by the following equations:

$$P_1 = C_1 V_1 = \pi \ell \left[C_1 R_1^2 - C_1 R_0^2 \right]$$
$$P_2 = C_2 V_2 = \pi \ell \left[C_2 R_2^2 - C_2 R_1^2 \right]$$
$$\vdots$$
$$P_j = C_j V_j = \pi \ell \left[C_j R_j^2 - C_j R_{(j-1)}^2 \right]$$
$$\vdots$$
$$P_n = C_n V_n = \pi \ell \left[C_n R_n^2 - C_n R_{(n-1)}^2 \right] \quad .$$

The parameter equation for the cylindrical composite wall is obtained by summation of the above P_j equations. The resulting equation

$$P = \pi \ell [-C_1 R_0^2 + C_1 R_1^2 - C_2 R_1^2 + C_2 R_2^2 \cdots - C_j R_{(j-1)}^2 + R_j^2 \cdots - C_n R_{(n-1)}^2 + C_n R_n^2]$$

was first presented in the last section as equation (4). Equation (2) is the single layer application of equation (4).

2. Technique for Determining Internal Radius

When a candidate material is evaluated as a new, $j=n$, layer, the external radius, $R_{(j-1)}$, of the previous layer must be recalculated. Since $R_{(j-1)}$ is also the internal radius of the new layer, the maximum allowable operating temperature of the new layer material will occur at the $R_{(j-1)}$ surface contact cylinder. Using this new external surface temperature for the previous layer, $R_{(j-1)}$, may be calculated using the heat flow equation for the previous layer. The previous layer heat flow equation is

$$q = \frac{2\pi k_{(j-1)} \ell \Delta t_{(j-1)}}{\ln(R_{(j-1)} / R_{(j-2)})} \tag{8}$$

The $R_{(j-2)}$ is the internal radius of the previous layer and $k_{(j-1)}$ and $\Delta t_{(j-1)}$ are respectively its conductivity and temperature drop. The temperature drop equals the difference between the hot surface temperature of the previous layer and the maximum allowable operating temperature of the new layer candidate material.

The internal radius of the new candidate layer is obtained by solving equation (8) for $R_{(j-1)}$. The resulting equation is

$$R_{(j-1)} = R_{(j-2)}\, e^{\left[(2\pi k_{(j-1)} \ell \Delta t_{(j-1)})/q\right]} \quad . \tag{9}$$

3. Technique for Determining External Radius

To apply the cylindrical parameter equation, it is necessary to know the external radius R_j of a j layer composite wall.

In this problem, as shown in Fig. 2, the heat flow rate, q, the inner radius, $R_{(j-1)}$, the fluid temperature, t_c, the convection film coefficient, h, and the conductivity, k, of the candidate insulation material are known. The temperature, t_j, that occurs at radius R_j as well as the R_j are unknown; therefore, two equations are required for solution.

For a steady state heat balance, the same heat flow rate, q, must pass through the j layer material and the fluid film in contact with the surface generated by R_j. The temperature t_j may be eliminated by addition of temperature difference equations for the insulation material and the fluid film.

Equation (8) can be rewritten to obtain the material, temperature difference as

$$t_{(j-1)} - t_j = \frac{q}{\left[\dfrac{2\pi k \ell}{\ln\left(R_j/R_{(j-1)}\right)}\right]} \quad . \tag{10}$$

The temperature difference across the fluid film, by Newton's law of cooling, is given by the equation:

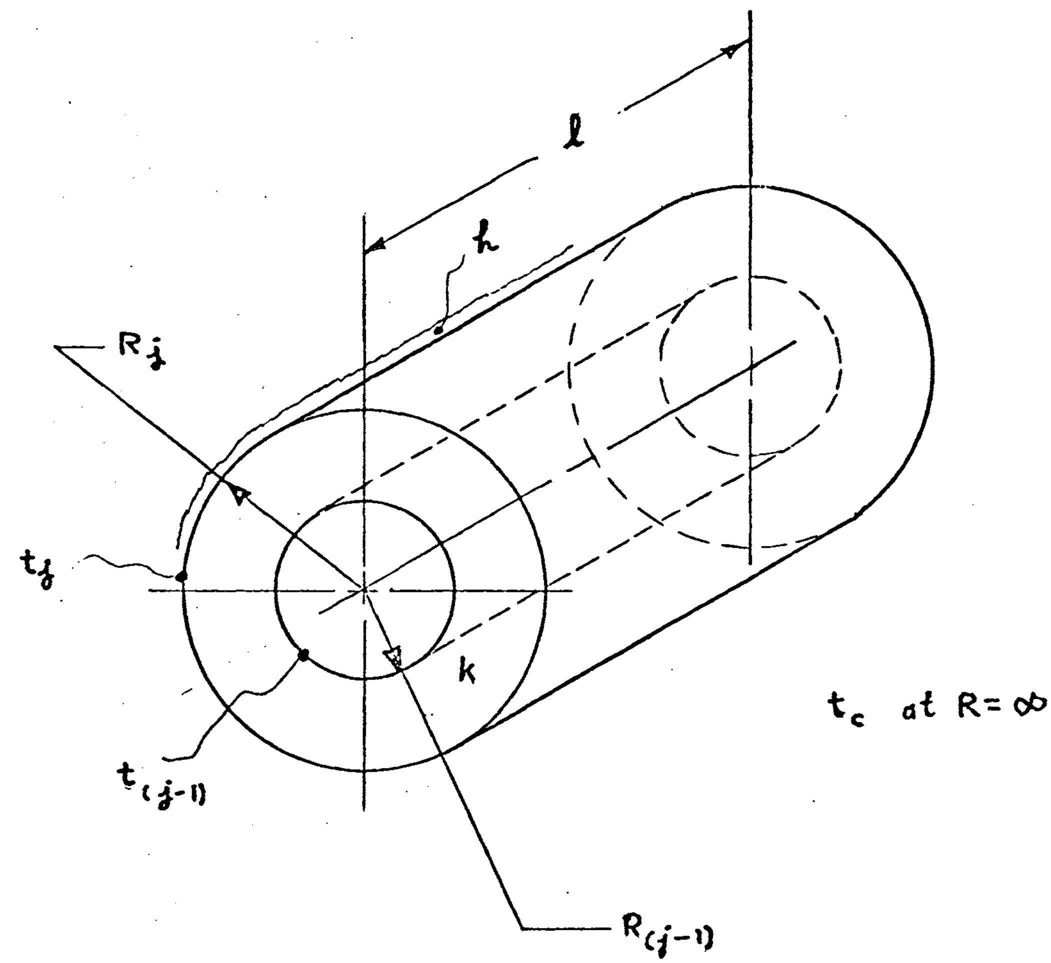


FIG. 2 - SKETCH OF RADIUS SELECTION PROBLEM

$$t_j - t_c = \frac{q}{2\pi R_j \ell h} \;. \tag{11}$$

By addition of equations (10) and (11) and rearranging the sum, the equation,

$$q\left[\frac{1}{k}\left(\ln R_j - \ln R_{(j-1)} + \frac{1}{hR_j}\right)\right] - 2\pi\ell\left[t_{(j-1)} - t_c\right] = 0, \tag{12}$$

is obtained.

Since q is known, R_j is the only unknown and its value can be determined. However, equation (12) is a transcendental equation in R_j, and a trial and error or an iteration technique must be used to solve it for the desired value of R_j.

Newton's iteration technique (3) may be used to solve for R_j. If R_{j_n} is the value of R_j after n iterations and if $f(R_{j_n})$ equals the value of the left side of equation (12), the equation for improving the accuracy of R_j is given by the equation,

$$R_{j(n+1)} = R_{j_n} - \frac{f(R_{j_n})}{f'(R_{j_n})} \tag{13}$$

where: $f'(R_{j_n}) = q\left[\dfrac{1}{kR_{j_n}} - \dfrac{1}{hR_{j_n}^2}\right]$.

The iteration process must be started with an initial value of R_{j_0} that will assure convergence to the desired value of R_j. When equation (12) is evaluated for q as a function of R_j, q increases from zero at $R_j = 0$, to a maximum at the critical radius $(R_j = k/h)$, and then decreases as R_j is further increased.

Figure 3 illustrates the variation of q with R_j for two different values of k, and shows the intersection of these two curves with two arbitrary values of maximum allowable heat loss, q_A. For convenience of presentation, h, and $R_{(j-1)}$, are set equal to unity; ℓ is set equal to $1/2\pi$; and $[t_{(j-1)} - t_c]$ is set equal to 1000^0F. With $R_{(j-1)}$ set equal to unity, the abscissa of Fig. 3 represents the radius ratio, $R_j/R_{(j-1)}$, as well as R_j. It is noted that curves for all values of k will pass through the point on the unity radius ratio line where all heat loss is due to convection, q_h. The $k = 1/2$ and $k = 2$ curves are respectively typical of curves which peak to the left and right of this point.

The solution obtained by the intersections of q_A lines and k curves at radius ratios of less than unity must be avoided as they represent negative layers thicknesses. Since the upper q_A value is greater than q_h, no solution is required. This example notes the necessity for initially calculating the convection heat loss from the uninsulated cylinder. When q_A is less than q_h, the intersections of a q_A line and k curve is a desired solution. In Fig. 3 the right hand intersection of the lower q_A line and the $k = 1/2$ curve is an example of a desired solution.

From the above discussion it is seen that a desired solution for $q_A < q_h$ will be obtained by setting R_{j_o} to a value greater than the critical radius k/h.

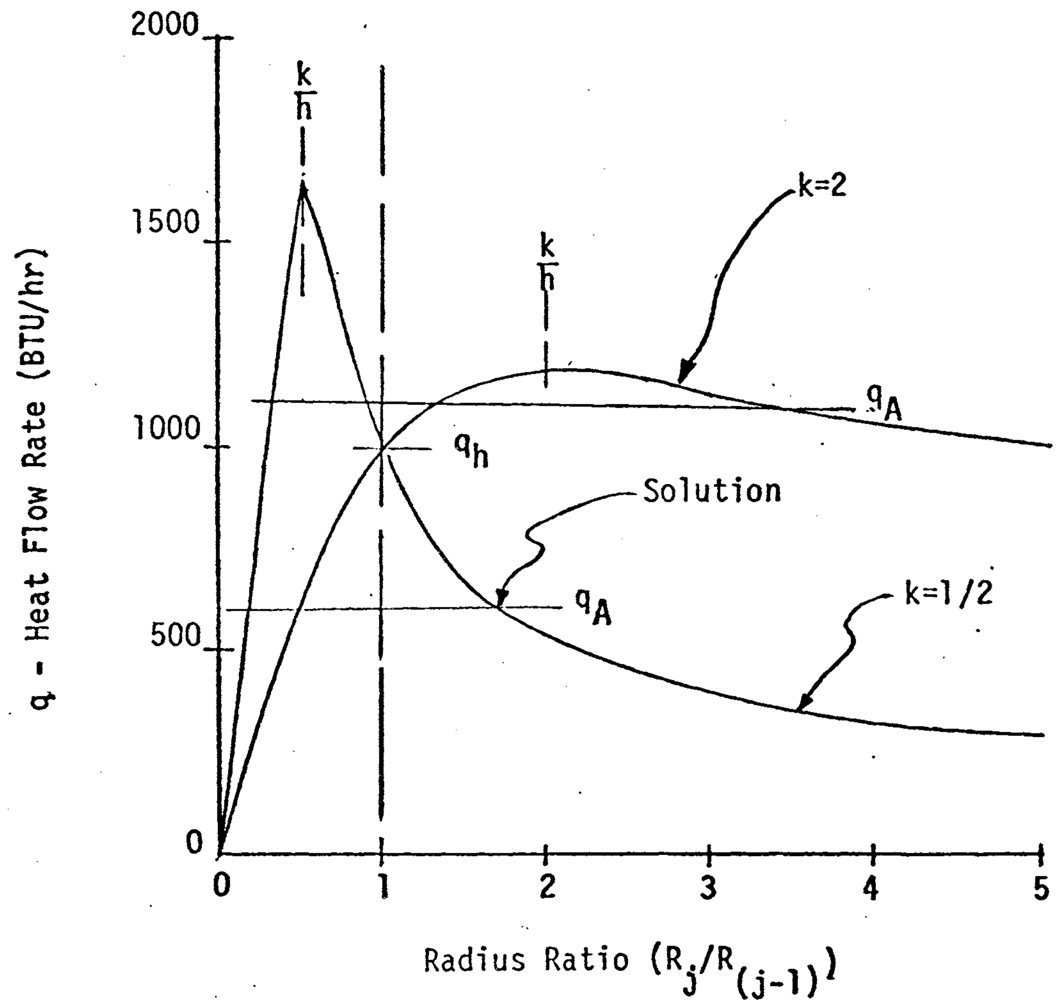


FIG. 3 - INITIAL VALUE SELECTION FOR ITERATION TECHNIQUE

RESULTS

The analyses methods and techniques described in the previous two sections resulted in the writing of two 360/50 computer programs in the Fortran IV language. Listings of the flat and cylindrical programs are presented in Appendices I and II respectively. Descriptive comment cards were inserted in each program deck. These comment cards describe input requirements and introduce major sections of analyses.

The material in this section is presented as a guide to users of the programs. Discussions of the input, output, and the solution to sample problems are presented for each program. Sample problems were solved using a material array containing data for 47 materials. The material data was obtained by a survey of insulation manufacturers.

A. FLAT WALL PROGRAM

1. Input

Figure 4 describes the data cards required with the flat wall program. All data cards are placed between a /DATA card and a /END card. This data pack is placed at the end of the program deck. Figure 4 shows the data cards in their proper sequence.

The Fortran name of each data item to be punched on a card are listed in order across the top of the card. The units for each data item are given below their Fortran names, and the Format to which the data are to be punched is given on each card.

The first data card specifies the size of the material array.

Each material data array card specifies, in order, the maximum allowed operating temperature, conductivity, cost per unit volume, and

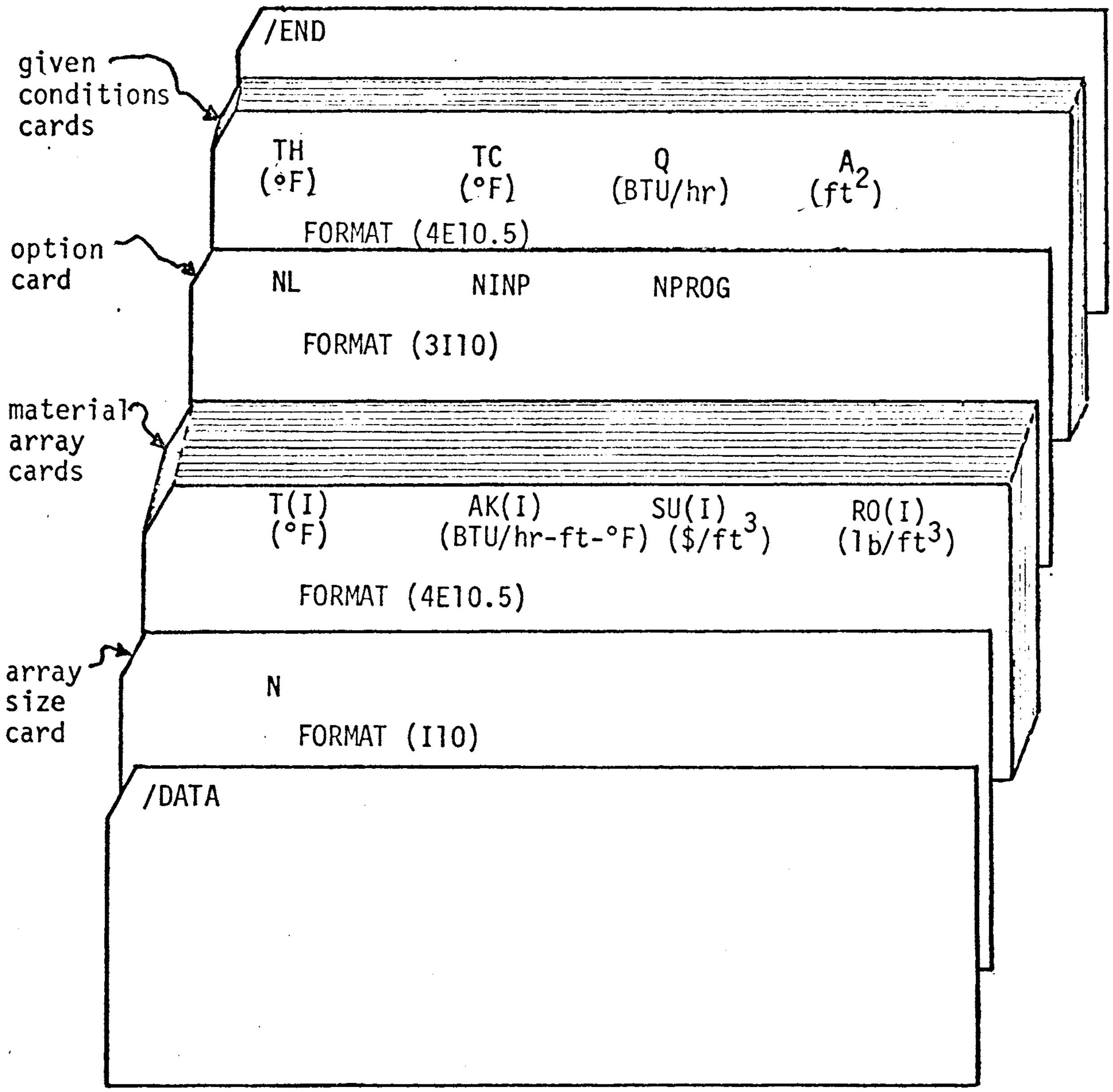


FIG. 4 - FLAT WALL DATA CARD

density of a particular material. All data for a particular material are given the same subscript by the computer. Subscripts are assigned one through N in the order that the cards are read.

The option card controls the number of layers at which optimization is to stop, print out of the material array, and the type of solution(s) to be found.

The material array is printed or not printed when NINP is given a value of two or one respectively. Given conditions are evaluated for minimum cost, weight, volume, or all three of these in order, when NPROG is assigned values of 1, 2, 3, or 4 respectively.

The hot and cold surface temperatures of the insulation wall, the heat flow rate through the wall, and the surface area of the wall are punched in order on a given conditions card. As many sets of given conditions as desired may be evaluated during one computer run.

2. <u>Output</u>

The program output presents three groups of information in order as follows: the material array; the problem statement; and the solution(s) to the problem statement. A material array output is shown in Fig. 5. As mentioned in a previous paragraph, the material array need not be printed. The problem statement and solutions are always printed.

The problem statement output describes the given conditions and required solution(s). Two problem statements are shown in Fig. 6. Figure 6 - a requires solution(s) to only one of three possible optimizations. Figure 6 - b requires that all three optimizations be made.

The computer program determines, in numerical order, the optimum wall configurations for all numbers of layers until the optimum number

THE MATERIAL DATA IS AS FOLLOWS:

MATERIAL NO.	MTL. TEMP. LIMIT DEG. F	CONDUCTIVITY BTU/(HR-FT-F)	COST $/(FT-SQ)	DENSITY LB./(CU.-FT.)
1	350.0	0.03500	0.540	0.60
2	350.0	0.03300	0.675	0.75
3	350.0	0.03000	0.900	1.00
4	350.0	0.02670	1.350	1.50
5	350.0	0.23400	1.800	2.00
6	450.0	0.04200	0.660	3.00
7	350.0	0.03800	0.770	3.50
8	450.0	0.03600	0.930	4.25
9	450.0	0.03100	1.320	6.00
10	450.0	0.02800	1.760	8.00
11	450.0	0.02700	1.980	9.00
12	350.0	0.02800	0.720	1.50
13	350.0	0.02420	1.010	2.25
14	350.0	0.02360	1.350	3.00
15	350.0	0.02220	1.910	4.25
16	220.0	0.02250	15.100	5.50
17	600.0	0.03600	3.960	11.00
18	1200.0	0.41600	3.960	11.00
19	1600.0	0.05750	3.990	21.00
20	1600.0	0.05750	3.990	21.00
21	1900.0	0.05000	2.200	11.00
22	2300.0	0.10400	10.400	3.00
23	2300.0	0.08900	13.800	4.00
24	2300.0	0.07400	20.800	6.00
25	2300.0	0.06750	27.700	8.00
26	2300.0	0.06330	34.460	10.00
27	2300.0	0.06000	41.600	12.00
28	2300.0	0.05250	48.500	14.00
29	2300.0	0.05400	62.300	18.00
30	2300.0	0.05160	83.200	24.00
31	2300.0	0.07900	4.000	4.00
32	2300.0	0.07500	6.000	6.00
33	2300.0	0.06500	8.000	8.00
34	2300.0	0.05800	12.000	12.00
35	2300.0	0.05500	18.000	18.00
36	2300.0	0.10000	12.000	6.00
37	2300.0	0.07600	9.600	4.00
38	2300.0	0.06500	14.400	6.00
39	2300.0	0.05830	55.700	12.00
40	2300.0	0.08170	18.500	16.00
41	300.0	0.02600	2.400	9.00
42	170.0	0.02000	12.800	2.00
43	212.0	0.02500	6.250	2.50
44	212.0	0.02670	17.500	7.00
45	212.0	0.03170	30.000	12.00
46	212.0	0.03660	47.500	19.00
47	200.0	0.05830	0.280	25.00

FIG. 5 - MATERIAL ARRAY OUTPUT

```
    SURFACE AREA IN SQ.-FT. (A)=          1.0
    HEAT TRANSFER RATE IN BTU/HR (Q)=       100.0
    HOT SURFACE TEMP. IN DEG. F (TH)= 2200.0
    COLD SURFACE TEMP. IN DEG. F (TC)=  100.0
    NUMBER OF MATERIALS TO BE EVALUATED (N)=47
    MAXIMUM NUMBER OF LAYERS TO BE USED (NL)= 3

FIND:
    THE LOWEST WEIGHT WALL
```

(a) Single option Problem Statement

```
    SURFACE AREA IN SQ.-FT. (A)=          1.0
    HEAT TRANSFER RATE IN BTU/HR (Q)=       100.0
    HOT SURFACE TEMP. IN DEG. F (TH)=  500.0
    COLD SURFACE TEMP. IN DEG. F (TC)=  100.0
    NUMBER OF MATERIALS TO BE EVALUATED (N)=47
    MAXIMUM NUMBER OF LAYERS TO BE USED (NL)= 4

FIND:
    THE LOWEST COST WALL
    THE LOWEST WEIGHT WALL
    THE LEAST VOLUME WALL
```

(b) Multi-option Problem Statement

FIG. 6 - FLAT WALL PROBLEM STATEMENT

of layers is found or until the layer limit (NL) is reached. If the optimum number of layers is less than the layer limit, the optimum layer solution is repeated until NL solutions have been printed. Figure 7 shows the solutions for a minimum weight optimization with a specified layer limit of three.

The first three of the seven columns of data, that follow each weight-of-wall print, specify the design of the wall. The material numbers, thicknesses, and hot surface temperatures of each layer are specified. The material numbers refer to locations of the material data in the material array. The last four columns data are the same material properties found in the material array. The material data is reprinted with the solutions for user convenience. The solution(s) output format for cost and volume optimizations are similar to the one shown for weight.

It is noted that Fig. 7 shows that a three layer wall of less weight than the optimum two layer wall was not found; therefore, the two layer solution was reprinted to satisfy the three solution prints required by the layer limit.

When the problem statement requires the solutions of all three optimizations, the solutions for minimum cost, weight, and volume are printed in order for a single layer wall. The process is continued with the three solutions for a two layer wall, etc., until the layer limit is reached. A multi-option problem statement and a portion of the solution set is shown in Fig. 8. The material data portion of the solutions output is not shown in Fig. 8.

The program has a provision for rejecting unrealistic problems. The statement "NO SOLUTION" is printed if the materials array contains

THE MINIMUM WEIGHT 1 LAYER WALL
HAS A TOTAL WEIGHT OF 0.6384E 01 POUNDS

MATERIAL NO.	THICKNESS INCHES	HOT SURFACE TEMP. DEG. F	MTL. TEMP. LIMIT DEG. F
37	19.15198	2200.0	2300.0

THE MINIMUM WEIGHT 2 LAYER WALL
HAS A TOTAL WEIGHT OF 0.5668E 01 POUNDS

MATERIAL NO.	THICKNESS INCHES	HOT SURFACE TEMP. DEG. F	MTL. TEMP. LIMIT DEG. F
37	16.84462	2200.0	2300.0
1	1.06260	353.0	353.0

THE MINIMUM WEIGHT 2 LAYER WALL
HAS A TOTAL WEIGHT OF 0.5668E 01 POUNDS

MATERIAL NO.	THICKNESS INCHES	HOT SURFACE TEMP. DEG. F	MTL. TEMP. LIMIT DEG. F
37	16.84462	2200.0	2300.0
1	1.06260	353.0	353.0

COST $/(CU.-FT.)	DENSITY LB./(CU.-FT.)	CONDUCTIVITY BTU/(HR-FT-F)
0.540	4.00	0.07600

COST $/(CU.-FT.)	DENSITY LB./(CU.-FT.)	CONDUCTIVITY BTU/(HR-FT-F)
0.540	4.00	0.07600
0.675	0.60	0.03500

COST $/(CU.-FT.)	DENSITY LB./(CU.-FT.)	CONDUCTIVITY BTU/(HR-FT-F)
0.540	4.00	0.07600
0.675	0.60	0.03500

FIG. 7 - FLAT WALL WEIGHT SOLUTION OUTPUT

```
          SURFACE AREA IN SQ.-FT. (A)=         1.0
          HEAT TRANSFER RATE IN BTU/HR (Q)=      100.0
          HOT SURFACE TEMP. IN DEG. F (TH)=   500.0
          COLD SURFACE TEMP. IN DEG. F (TC)=  100.0
          NUMBER OF MATERIALS TO BE EVALUATED (N)=47
          MAXIMUM NUMBER OF LAYERS TO BE USED (NL)= 4

      FIND:
          THE LOWEST COST WALL
          THE LOWEST WEIGHT WALL
          THE LEAST VOLUME WALL


         THE MINIMUM COST   1 LAYER WALL
         HAS A COST OF 0.4400E 00 DOLLARS

 MATERIAL              THICKNESS            HOT SURFACE
      NO.                 INCHES            TEMP. DEG. F
       21                2.40000                   500.0


         THE MINIMUM WEIGHT   1 LAYER WALL
         HAS A TOTAL WEIGHT OF 0.1216E 01 POUNDS

 MATERIAL              THICKNESS            HOT SURFACE
      NO.                 INCHES            TEMP. DEG. F
       37                3.64800                   500.0


         THE MINIMUM VOLUME   1 LAYER WALL
         HAS A TOTAL VOLUME OF 0.1440E 00 CUBIC FEET

 MATERIAL              THICKNESS            HOT SURFACE
      NO.                 INCHES            TEMP. DEG. F
       17                1.72800                   500.0


         THE MINIMUM COST   2 LAYER WALL
         HAS A COST OF 0.1520E 00 DOLLARS

 MATERIAL              THICKNESS
      NO.                 INCHES
       21                0.300
        6                1.
```

FIG. 8 - FLAT WALL MULTI-OPTION SOLUTION OUTPUT

no material with an allowable operating temperature greater than the hot surface temperature of the problem statement.

3. Solution Evaluation

Inspection of the flat wall design equation (3) shows that the parameter being minimized is directly proportional to the square of the surface area and inversely proportional to the heat flow rate. Introducing the proportionality constant D, the design equation may be rewritten as:

$$P = DA^2/q \,, \tag{14}$$

The selection of layer materials, the temperature distribution in the selected layers, and therefore D are dependent only upon the type of optimization, the content of the material array, and the exterior surface temperature of the wall.

Review of the thickness equations (6) shows that the thickness of a wall layer is proportional to surface area and inversely proportional to heat flow rate. By introducing the proportionality constant B, layer thickness may be expressed:

$$x = B\,A/q \;; \quad \text{where } B = K\Delta t. \tag{15}$$

The computer program may be used to find the solution(s) to any particular problem statement. However, if the same material array is used, the solutions to all problems which have the same hot and cold temperatures will be proportional to each other. The solutions to a single problem statement can be used to calculate D and B constants. Equations (14) and (15) can then be used to define optimum walls with different areas and heat flow rates. Figure 9 is a tabulation of D and B values for walls with hot and cold temperatures of 2200°F

MINIMUM COST

No. of layers	one	two	three	four
last material	31	22	6	1
D	663.6	292.8	264.0	261.8
B_1	1990.8	284.4	284.4	284.4
B_2		1080.0	870.0	870.0
B_3			176.4	48.9
B_4				106.3
B TOT	1990.8	1364.4	1330.8	1471.4
% Reduction	0	55.8	60.2	60.6

MINIMUM WEIGHT

No. of layers	one	two
last material	37	1
D	638.4	566.8
B_1	1915.2	1684.5
B_2		106.3
B TOT	1915.2	1790.8
% Reduction	0	11.0

MINIMUM VOLUME

No. of layers	one	two	three	four
last material	30	11	15	42
D	108.4	99.8	98.6	98.4
B_1	1300.3	1083.6	1083.6	1083.6
B_2		113.6	32.4	32.4
B_3			66.6	48.0
B_4				16.8
B TOT	1300.3	1197.0	1182.6	1180.8
% Reduction	0	26	17	17

NOTE: 1 - Material data from Fig. 5.
2 - Surface temperatures 2200° F and 100° F

FIG. 9 - FLAT WALL PROPORTIONALITY CONSTANTS

and 100°F. The material array shown in Fig. 5 was used for the computer run that resulted in Fig. 9. Proportionality constants are given for minimum cost and volume for walls of one through four layers and minimum weight walls of one and two layers. The B values with number subscripts are for calculating individual layer thicknesses. The last row of data under each optimization is the per cent saving with respect to the single layer minimized parameter.

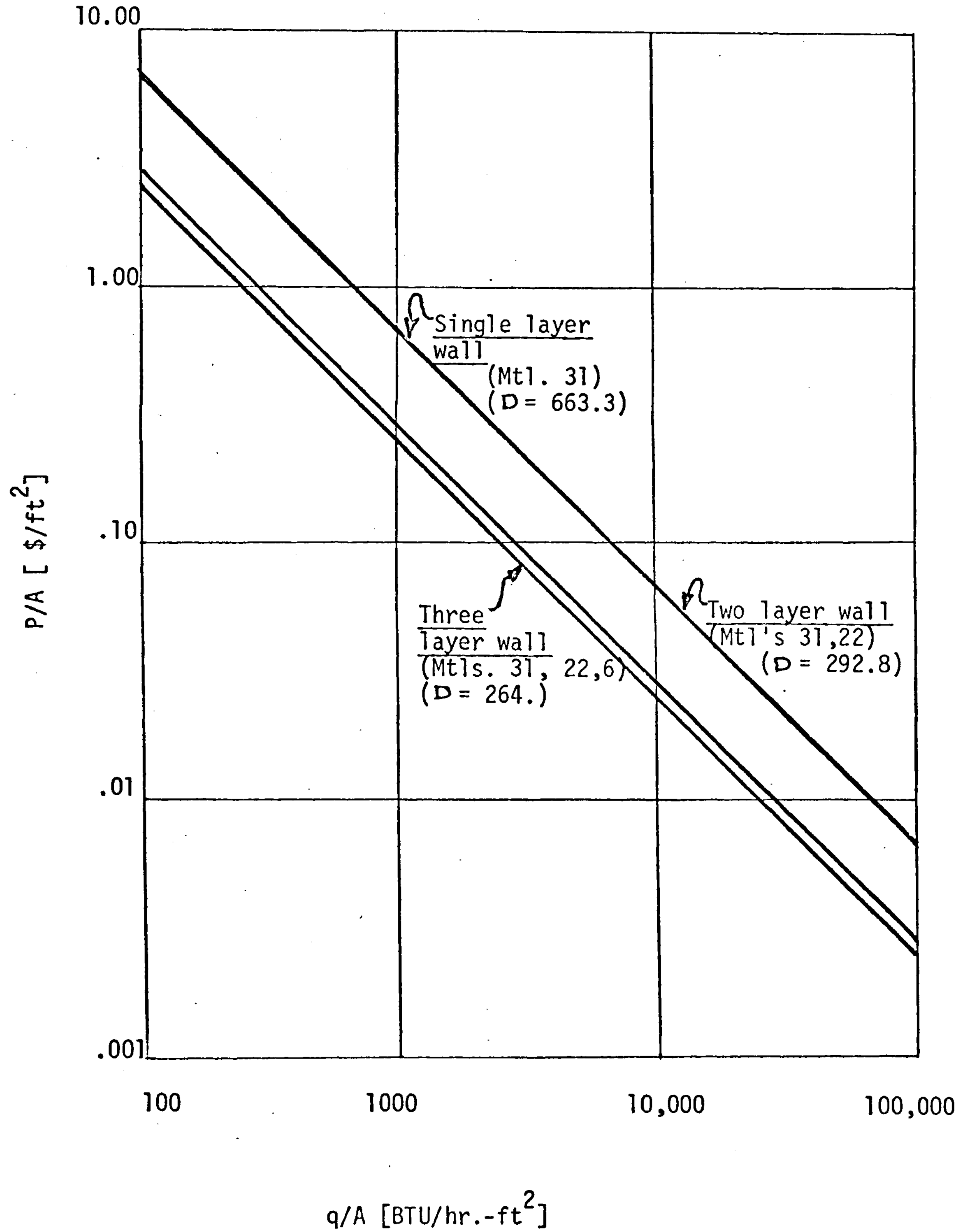
When the computer results are to be used as part of a more complex study, graphical presentations may be convenient. It is noted that any of the relationships involving D and B graph as a straight line on logarithmic graph paper. Figure 10 is a sample logarithmic graph showing the minimum cost per unit area versus heat flow rate per unit area for 1, 2, and 3 layer walls. The D constants of the walls were taken from Fig. 9.

B. CYLINDRICAL WALL PROGRAM

1. Input

Figure 11 describes the data cards required with the cylindrical program. The data pack is placed at the end of the program deck. With the exception of the given-conditions cards, the cylindrical wall data pack is identical to the flat wall data pack. Cards which are identical are used in exactly the same way, and the explanations given for the flat wall program may be applied. It is noted that the same set of material array cards may be run with either program.

Each cylindrical given-conditions card specifies, in the order shown on the card, the following:



NOTES:

1) Surface temperatures 2200°F and 100°F.
2) Material Array of Fig. 5.

FIG. 10 - FLAT WALL MINIMUM COST OPTIMIZATION

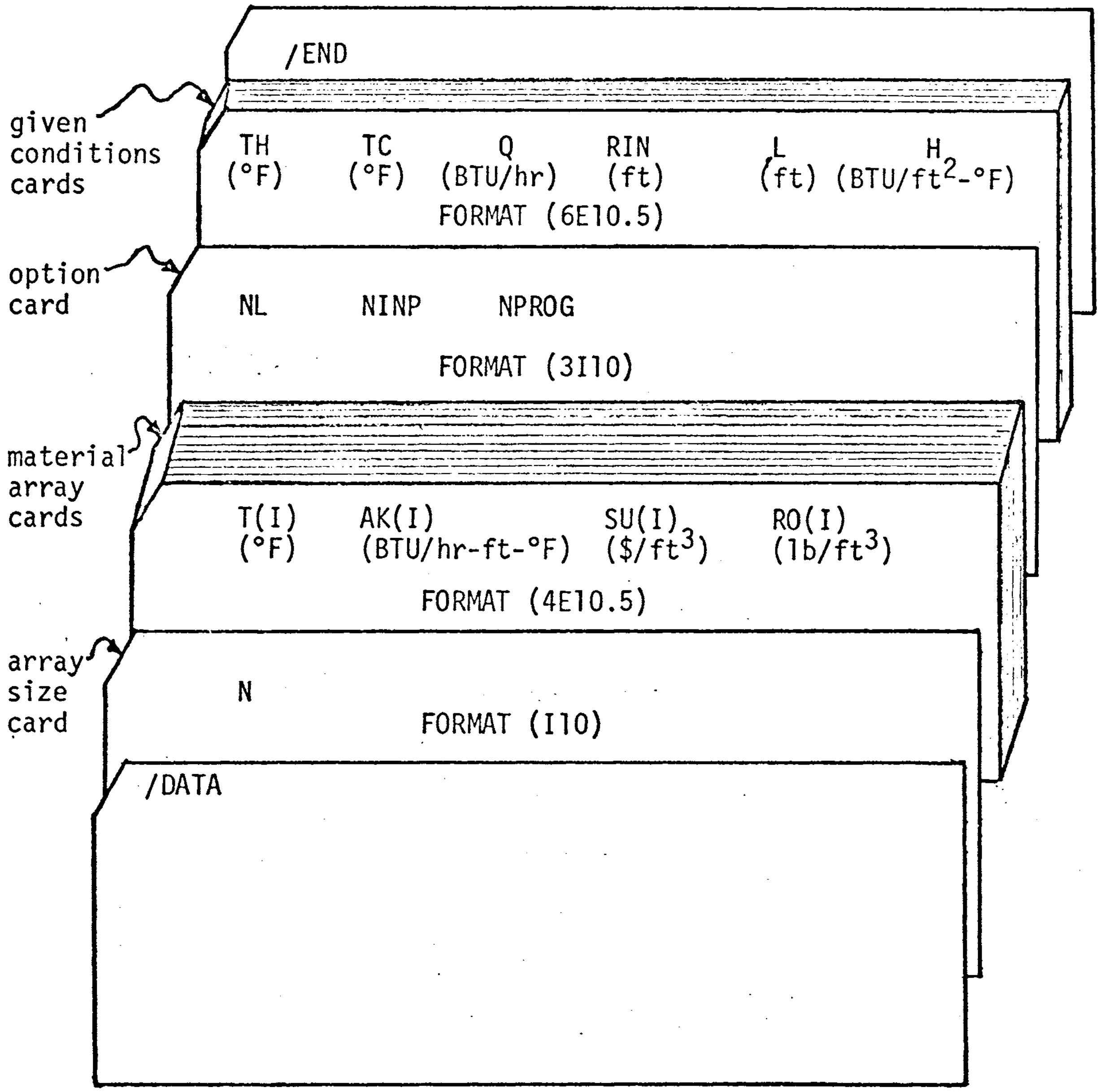


FIG. 11 - CYLINDRICAL WALL DATA CARDS

1 - The temperature at the internal hot surface of the insulation wall.

2 - The temperature of cold fluid in contact with the external surface of the insulation wall.

3 - The heat flow rate through the insulation wall.

4 - The internal radius of the insulation wall.

5 - The length of the cylinder.

6 - The convective film coefficient at the external surface.

As many given conditions-cards as desired may be used during a single computer run.

2. Output

The form of the cylinderical program output is similar to the form used for the flat program. The material array output format is the same for both programs, and Fig. 5 is an example. Again, material array output is a user option.

As with the flat wall program, problem statements may be used to request the solutions to anyone or all three of the optimization options. A sample three option problem statement is shown in Fig. 12. The order in which the solutions are printed is the same as was described for the flat program; but, the format is slightly different. Figure 13 shows a portion of a three option solution set output.

Each solution output presents: the number of layers in the wall; the minimum cost, weight or volume of the wall; the design data; and the materials data. The first three columns of data specify the wall design. The material number refers to the location of the layer material in the

```
HEAT TRANSFER RATE IN BTU/HR (Q)=    1000.0
LENGTH OF CYLINDER IN FT. (L)=    1.000
RADIUS OF CYLINDER IN FT. (RIN)= 1.000
SURFACE CONDUCTANCE IN BTU/HR-FT. SQ.-F   (H)= 5.000
SURFACE TEMPERATURE OF CYLINDER DEG. F (TH)= 2200.0
FLUID TEMPERATURE IN DEG. F (TC)=   100.0
NUMBER OF MATERIALS TO BE EVALUATED (N)=47
MAXIMUM NUMBER OF LAYERS TO BE USED (NL)= 4

FIND:
   THE LOWEST COST WALL
   THE LOWEST WEIGHT WALL
   THE LEAST VOLUME WALL
```

FIG. 12 - CYLINDRICAL WALL PROBLEM STATEMENT

THE MINIMUM COST 1 LAYER WALL
HAS A COST OF 0.8733E 02 DOLLARS

MATERIAL NO.	LARGE RADIUS OF LAYER FT.	TEMP. AT LARGE RADIUS DEG. F	COST $/(CU.-FT.)	DENSITY LB./(CU.-FT.)	CONDUCTIVITY BTU/(HR-FT-F)
31	2.82011	111.3	4.000	4.00	0.07900

THE MINIMUM WEIGHT 1 LAYER WALL
HAS A TOTAL WEIGHT OF 0.7973E 02 POUNDS

MATERIAL NO.	LARGE RADIUS OF LAYER FT.	TEMP. AT LARGE RADIUS DEG. F	COST $/(CU.-FT.)	DENSITY LB./(CU.-FT.)	CONDUCTIVITY BTU/(HR-FT-F)
37	2.71065	111.7	9.600	4.00	0.07600

THE MINIMUM RADIUS 1 LAYER WALL
HAS A TOTAL VOLUME OF 0.8987E 01 CUBIC FEET

MATERIAL NO.	LARGE RADIUS OF LAYER FT.	TEMP. AT LARGE RADIUS DEG. F	COST $/(CU.-FT.)	DENSITY LB./(CU.-FT.)	CONDUCTIVITY BTU/(HR-FT-F)
30	1.96520	116.2	83.200	24.00	0.05160

THE MINIMUM COST 2 LAYER WALL
HAS A COST OF 0.2360E 02 DOLLARS

MATERIAL NO.	LARGE RADIUS OF LAYER FT.	TEMP. AT LARGE RADIUS DEG.	COST $/(CU.-FT.)	DENSITY LB./(CU.-FT.)	COND
31	1.16048		4.000		
21	2.03278				

FIG. 13 - CYLINDRICAL WALL MULTI-OPTION SOLUTION OUTPUT

material array. The large radius of each layer is given in feet. The small radius of the first layer is given in the problem statement. The third column of data lists the temperatures at the large radii of the layers. The temperature at the small radius of the first layer is given in the problem statement. The temperature drop across the fluid film may be found by subtracting the fluid temperature of the problem statement from the external surface temperature of the external layer. The maximum allowable operating temperature of a layer is equal to the large radius temperature of the previous layer. The three right hand columns are a repeat of material array data. The program has provisions for the rejection of unrealistic problems. The statement "NO SOLUTION" is printed if the material array does not contain materials with maximum allowable operating temperatures greater than the hot temperature of the problem statement. When heat convection from the internal surface of the wall is equal to or less than the specified heat flow rate, the statement "NO INSULATION IS REQUIRED. HEAT CONVECTION FROM THE CYLINDER IS (the value in BTU/HR.)" is printed.

If the external radius of the optimum single layer wall is found to be excessively large, the statement "THE ALLOWED HEAT FLOW RATE IS TOO SMALL" is printed. The judgment is set at a value of XX=7.0 in the large radius equation, i.e.,

```
        RBIG = (EXP(XX) * RSML + RSML)/2.0,
where   XX=6.28 * CON * L * DT/Q.
```

This judgment may be changed by substituting a value different than 7.0 in the statements on lines 68 and 99 of the program listing (See Appendix II).

The program contains an abort provision in the event that the Newton's iteration technique fails to converge on a large radius value within 50 iterations. In this event the statement "PROGRAM STOPPED ---50 ITERATIONS ON RADIUS" is printed. This abort condition has not occured to this time.

When a series of problems, which are similar except for the heat flow rate, are solved, the material selected for a particular layer may change as the heat flow rate changes. This condition can result from round off errors in the calculated values of two radii of a layer and/or from the finite difference check on the convergence of the Newton's iteration technique. When the true layer thickness is very small, the computer may, in error, calculate a negative layer thickness. A program check (line 117 of the listing) rejects zero and negative layer thicknesses. Since layer thicknesses increase as the heat flow rate decreases, layer materials which were rejected at high flow rates are used at lower flow rates.

3. Solution Evaluation

Inspection of the cylindrical wall parameter equation (4), shows that the parameter being minimized is directly proportional to the length of the cylinder. Other simple algebraic relationships between variables are not immediately apparent; however, relationships may be shown with tables and graphs. The required data can be easily obtained by using the computer program.

The computer program was used to determine the effect of changing the amount of heat flow rate upon the design of a cylindrical wall with the otherwise constant set of given conditions shown in Fig. 12. Heat flow rate inputs of 1000, 1150, 1500, 2000, 3000, 4000, 6000, and 10000 BTU/HR. were used. The data taken from the computer runs are tabulated in Appendix III. Trial graphs of the program results were made on linear, logarithmic, and semilogarithmic coordinate paper. Semilogarithmic coordinates were found to be the best means of demonstrating the effects of heat flow rate. The values of the variables of interest may be read on linear scales while the logarithmic scale allows a large range of heat flow rate to be covered.

Semilogarithmic graphs of the results of the cost, weight, and volume optimizations are shown in Fig's. 14, 15 and 16 respectively. Each figure has four graphs. Starting from the top, the first graph shows minimum cost, weight, or volume. The second graph shows the percent savings that results from the use of a multi-layer wall in the place of a single layer wall. The next graph shows the external surface temperatures of the walls, and the last graph shows the external radii of the walls. Each curve, shown on a graph, is for the optimum wall with the number of layers specified by the callout. Curves are not shown for walls for which the data falls coincident upon the curves of walls of a lesser number of layers.

Cost optimization program results were printed for 1, 2, 3, and 4 layer walls for heat flow rates from 1,000 to 4,000 BTU/HR., and for 1 and 2 layer walls from 1,000 to 10,000 BTU/HR. Apparently the true thicknesses of the third and fourth layers are so small at high heat flow rates that they were calculated as negative values and were rejected. No four layer wall curves are shown on Fig. 14 because the data was near coincident with 3 layer wall data, and for the same

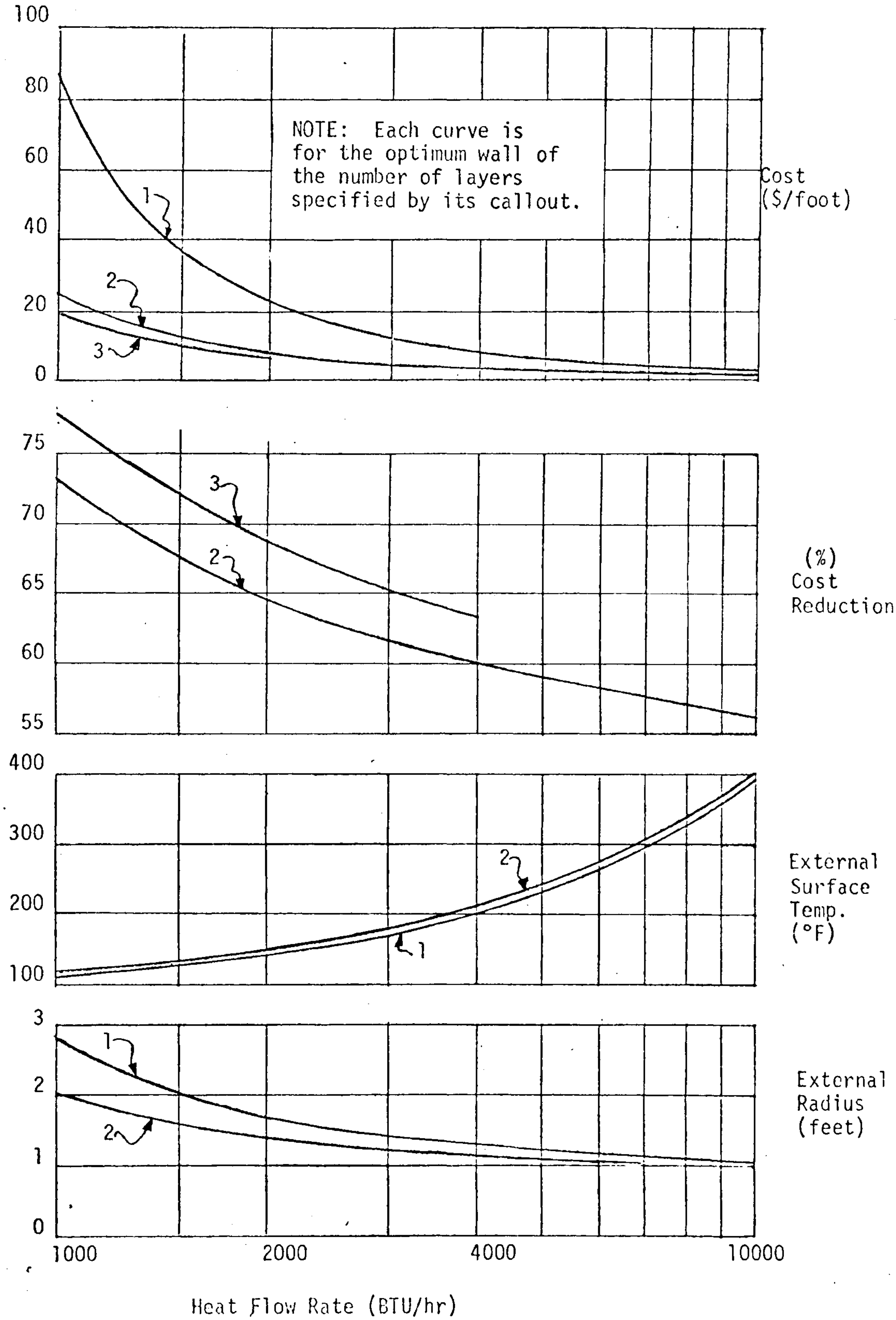


FIG. 14 - CYLINDRICAL WALL-MINIMUM COST OPTIMIZATION

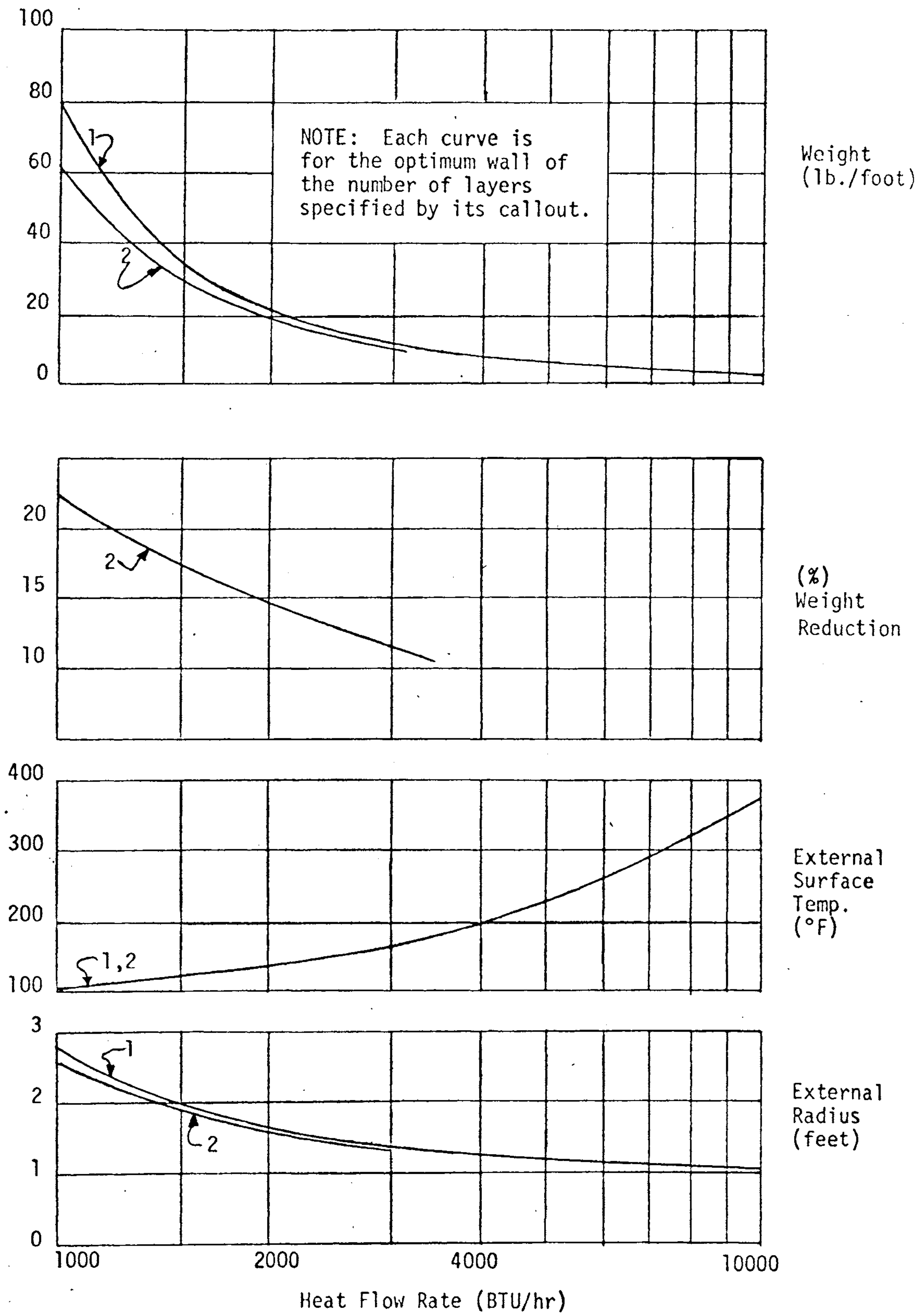


FIG. 15 - CYLINDRICAL WALL-MINIMUM WEIGHT OPTIMIZATION

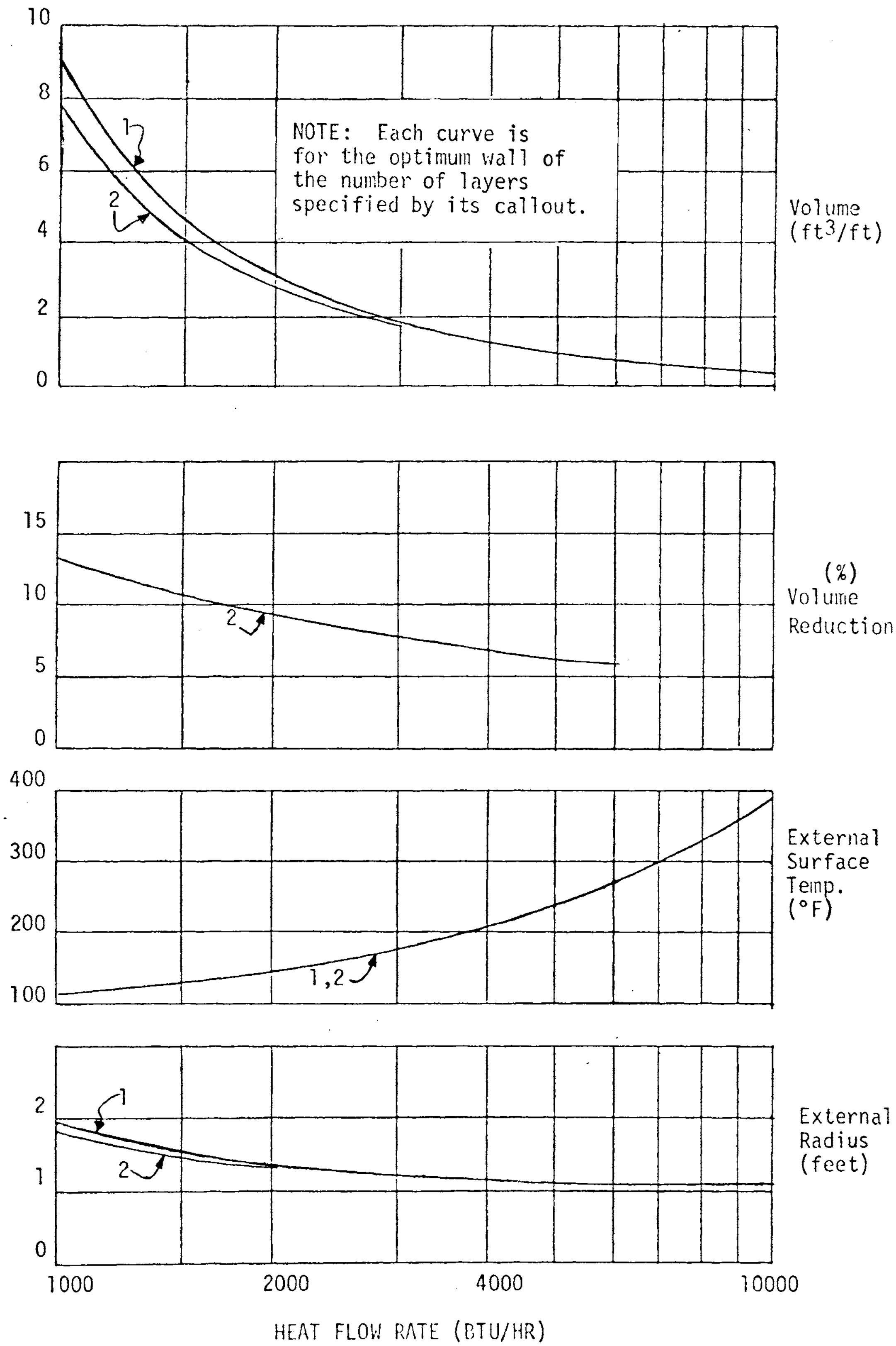


FIG. 16 - CYLINDRICAL WALL-VOLUME OPTIMIZATION

reason, 3 layer wall temperature and radius curves are not shown.

Results such as those shown in Fig. 14 can be used to find the minimum material cost for walls of different numbers of layers as a function of heat flow rate. This material cost data can be used in conjunction with other cost data as part of more complex cost studies. For example, the most economical insulation configurations for the plumbing in a power plant could be determined using computer material cost results in conjunction with data pertaining to installation, thermal energy, and depreciation costs. Results as shown in Fig. 14 might also be used to find the minimum overall cost of reducing the external surface temperature of pipe insulation to a specific value. For example, for the given conditions resulting in Fig. 14, suppose it is desired to reduce the external surface temperature to 200° F at minimum cost. In this case the temperature graph is entered at 200° F and it is found that single and double layer walls will allow about 4,000 BTU/HR. and 3600 BTU/HR of heat flow rates respectively. Entering the cost graph at 4,000 and 3,600 BTU/HR., the material costs of one and two layer walls are found to be approximately 8 and 4 dollars per running foot respectively. The choice between the one and two layer configuration would then be made upon the basis of installation labor cost. If more accurate values of material costs were desired, additional computer runs could be made using fine increments of heat flow rate around 4,000 and 3,600 BTU/HR. These results could then be plotted as cost versus temperature on linear coordinates.

Weight optimization results were printed for the single layer wall for the full range of heat flow rate inputs. Two layer results were obtained for flow rates from 1000 to 3000 BTU/HR. No three or four layer results were printed. The external surface temperatures for single and double layer walls were within one degree of each other; therefore, only the single layer temperature curve is shown in Fig. 15. Minimum weight data such as that presented in Fig. 15 can be used to solve directly for the minimum weight wall of one or two layers for a specific heat flow rate. The computer results might also be used as part of more complex studies. For example, an aerospace application might require a trade-off-study to minimize overall weight penalty due to heat loss and material weight.

The minimum insulation weight required to reduce the external wall surface temperature to a desired value can be determined directly. For example, Fig. 15 shows that a desired surface temperature of 200°F is obtained at 4000 BTU/HR., and that the minimum single layer insulation weight is approximately 7.7 pounds per running foot. In the transportation industry, studies might be performed to determine the effect of insulation weight upon fuel cost.

Minimum volume results were output for the single and the two layer walls from 1000 to 10,000 BTU/HR. The second layer of the two layer wall had material changes at 4,000 and 10,000 BTU/HR. Three and four layer results were not printed for the complete range of heat flow rates and a number of material changes occured. Only one and two layer results are shown in Fig. 16. The two layer curves are ended at 3000 BTU/HR. because of material changes at the higher heat flow rates. One and two layer temperature data were within one degree. Three and four layer data is near coincident upon the two layer data.

Minimum volume computer results may be used in analyses similar to those described for cost and weight. Minimum volume applications might arise in the aerospace and transportation industries or from consumer products industry where sales appeal can be improved by reducing volume.

DISCUSSION OF RESULTS

The initial objectives of this project were to devise methods for determining the minimum cost, weight, or volume for flat and cylindrical composite insulation walls and to write computer programs to accomplish these analyses. These objectives have been fulfilled, and the previous section serves as a guide in the use of the two resulting computer programs. The flat and cylindrical shapes were selected for analyses because of their frequent application.

Insulation data obtained by a survey of manufacturers was used to give a sense of reality to the results which were obtained to demonstrate the function of the computer programs. However, the present compilation of data is not considered to be sufficiently complete or detailed for effective use of the computer programs. The establishment of consistent and detailed methods for collecting, presenting, and filing data is recommended. Materials which are not normally considered to be insulations should not be overlooked. Material environmental limitations other than temperature should also be noted.

In addition to obtaining a current and comprehensive collection of material data, the user of the programs must exercise judgment to assure the accuracy of his results. The need for judgment stems from simplifying assumptions basic to the analyses and can be reduced by increasing the complexity of the computer programs. Inspection of the basic assumptions indicates that the most gain for the effort expended will result from changing the material conductivity assumption.

One implied assumption is that the unit volume cost of a material is constant. Since unit costs change with time, quantity, and configuration the constant unit cost assumption requires the user to know current costs and to have some forehand knowledge of the material quantity requirements by his problem statement.

It is felt that a change from the present cost assumption is not justified because different manufacturers use different methods for cost adjustment and because price breaks usually occur at fixed quantities.

Because the negligible contact resistance assumption yields conservative results, and because of the uncertainty and complexity involved when contact resistance is considered, it is felt that a change from the present assumption should have low priority. The cylindrical program problem statement requires the input of a fixed value for the convective film coefficient. This restriction requires initial judgment in the selection of a coefficient value and an evaluation of results to verify the selection. The cylindrical program can be modified to make the film coefficient a function of radius; but it is felt that this change should take second priority to changing the material conductivity assumption for both programs.

It is now assumed that the conductivity of each material is a constant. When this assumption is not true for the materials used with the program, the user must inspect conductivity data and select mean input values in keeping with his problem statement. He must also inspect the program results to verify his selections. This need for judgment can be greatly reduced by assuming that conductivity is a linear function of temperature.

The limitation of the constant conductivity assumption became apparent during inspection of material data collected from manufacturers. The conductivity of many materials varies greatly with temperature. In a number of cases manufacturers provided both summary and detail material data. The detail data presented conductivity versus temperature data. The summary data gave a specific value of conductivity. Usually the specific value was the mean conductivity effective between the maximum operating temperature and room temperature. Such a value is usable for single layer flat walls, but is not suitable for the analysis of composite walls. It is felt that the needs of the computer programs can best be met by the simple expedient of linarizing variable conductivity data. In cases where only one value of conductivity was given by the manufacturer, it was not certain whether the value was a constant or a mean, or whether only a limited amount of testing had been performed. When possible, these uncertainties should be eliminated.

A change to the linear conductivity assumption will require two changes to the material data array cards. The card item location now used for the constant conductivity input can be used to specify the value of conductivity at the hot temperature of the material. A data item specifying the rate of change of conductivity with temperature must be added to the card. The program logic must be changed in two ways. Firstly the conductivity of each candidate layer material must be calculated and substituted into the optimum value equation. Secondly, an optimum previous layer material must be selected along with each candidate layer material. The second requirement results from the fact that the hot temperature of a new layer becomes the cold temperature of the previous layer and thus changes the differential temperature upon which previous layer's conductivity was based.

The computer programs were designed for applications with heat flow from a hot source to a cold sink. If the candidate materials will not be damaged by exposure to the cold temperature of the problem statement, the programs may be used for problems where the heat flow into a cold sink is to be limited. For the cylindrical wall, the cold source must be on the external diameter of the insulation wall. Cold sink programs based upon the existing program can be written in which the cold temperature limitations of the materials are considered. When and if cold sink programs are written, spherical insulation should be considered, as cryogenic storage tanks are often spherical.

CONCLUSIONS AND RECOMMENDATIONS

The objectives of this project have been accomplished. Computer programs for optimizing flat and cylindrical composite insulation walls have been written, and their use has been demonstrated. The quality of results depends only upon the accuracy and completeness of the input material data.

It is suggested that the usefulness of the computer programs be increased by performing the two tasks that follow:

1. Establish detail procedures for collecting and filing material data.
2. Modify both programs to allow for linear change of conductivity with temperature.
3. Modify the cylindrical program to allow the film coefficient to change as a function of radius.
4. Write new programs for cryogenic applications.

REFERENCES

1. Wilson, Allen C., Industrial Thermal Insulation, McGraw-Hill Book Company, Inc., 1959.

2. Smith, Harvell M., "Pipe Insulation an Integral Part of Piping Systems," Air Conditioning Heating and Ventilating, May 1968.

3. Conte, S. D., Elementary Numerical Analysis, McGraw-Hill Book Company, Inc., New York, 1965.

APPENDIX I

LISTING OF FLAT WALL COMPUTER PROGRAM

```
/WAT4  MS120265,TIME=1,PAGES=10  D              MORATH BILL                        JOB 219
           DIMENSION T(50),AK(50),SU(50),RO(50),C(50),M(50),ARO(50)
           DIMENSION TT(50),AAK(50),CC(50),SUM(50),X(50),ASU(50)
           READ (1,1000) N
      C    N=NUMBER OF CARDS IN THE MATERIAL ARRAY (ONE CARD PER MATERIAL)
      C    INPUT MATERIAL ARRAY
           READ(1,1005) (T(I),AK(I),SU(I),RO(I),I=1,N)
      C    T(I)=MAXIMUM ALLOWED TEMP. OF MATERIAL (F)
      C    AK(I)=CONDUCTIVITY OF MATERIAL (BTU/(HR-FT-F)
      C    SU(I)=MTL.COST/VOLUME ($/(CU.-FT.)
      C    RO(I)=MTL. DENSITY (LB./(CU.-FT.)
           READ (1,1000) NL,NINP,NPROG
      C    NL=MAXIMUM NUMBER OF LAYERS TO BE ALLOWED IN THE FINAL RESULT
      C    NINP=INPUT A 2 FOR PRINT OUT OF MTL. ARRAY.  INPUT A 1 FOR PRINT NO .
      C    PRINT OUT
      C    NPROG INPUT: 1,2,3,4 FOR MIN. COST MIN. WT., MIN. THICKNESS, ALL 3.
           WRITE(3,2030)
           IF(NINP.GE.2) GO TO 65
           GO TO 67
        65 WRITE(3,1025)
      C    WRITE: MTL. ARRAY IF NPIN=1.
           WRITE(3,1026)(I,T(I),AK(I),SU(I),RO(I),I=1,N)
        67 CONTINUE
      C    INPUT PROBLEM STATEMENT
      8686 READ(1,1001) TH,TC,Q,A
      C    TH=HOT SURFACE TEMPERATURE (F)
      C    TC=COLD SURFACE TEMPERATURE (F)
      C    Q=HEAT FLOW RATE THRU WALL (BTU/HR)
      C    A=SURFACE AREA OF WALL (SQ.-FT.)
           WRITE(3,1020)
      C    WRITE: PROBLEM STATEMENT
           WRITE(3,1021) A,Q,TH,TC,N,NL
           GO TO (71,72,73,74 ),NPROG
        71 WRITE (3,1022)
           GO TO 2
        72 WRITE(3,1023)
           GO TO 2
        73 WRITE (3,1024)
           GO TO 2
        74 WRITE(3,1022)
           WRITE(3,1023)
           WRITE(3,1024)
         2 CONTINUE
           NLIND=NL
           DO 7676 MMM=1,NLIND
           NL=MMM
      C    SELECT OPTIMIZATION OPTION I.E.,NPROG=1,2,3,OR4
           IF(NPROG-4)4,3,3
         3 NXX=1
           GO TO 5
         4 NXX=NPROG
         5 CONTINUE
           GO TO (6,7,8),NXX
         6 DO 66 I=1,N
           C(I)=SU(I)
        66 CONTINUE
           GO TO 5
```

```
      7 DO 77 I=1,N
        C(I)=RO(I)
C       C(I)=DUMMY VARIABLE DEPENDING UPON NPROG=NXX=1.2,OR3
     77 CONTINUE
        GO TO 9
      8 DO 88 I=1,N
        C(I)=1.0
     88 CONTINUE
        GO TO 9
      9 CONTINUE
C       ESTABLISHING INITIAL VALUES FOR OPTIMIZATION:
        SUMCJ=10.0E6
        NOOEW=0
C       START OF OPTIMIZATION - THE LOOP DO 300 J=1,NL,DETERMINES
C       IF N OR N+1 LAYERS ARE OPTIMUM.
        DO 300 J=1,NL
        IF(J-1)10,10,20
     10 CKM=10.0E6
C       THE INNER LOOP DO 100 I=1,N, SELECTS THE MTL FOR THE HOT LAYER
        DO 100 I=1,N
        JJ=J
        IF(T(I)-TH)13,13,11
     11 IF(C(I)*AK(I)-CKM)12,12,100
     12 CKM=C(I)*AK(I)
        CC(J)=C(I)
        AAK(J)=AK(I)
        M(J)=I
        ASU(J)=SU(I)
        ARO(J)=RO(I)
        TT(J)=TH
        GO TO 100
     13 CONTINUE
        IF(I-N) 100,144,144
C       STATEMENT 888 INDICATES THAT NO MTL. IN THE ARRY WILL
C       OPERATE AT THE HOT TEMP. OF THE PROBLEM STATEMENT.
    144 IF(CKM=10.0E6)100,14,14
     14 IF(T(I).LT.TH) GO TO 888
    100 CONTINUE
        SUM(J)=AAK(J)*CC(J)*(TH-TC)
        GO TO 222
C       THE INNER LOOP DO 200 I= N, SELECTS THE OPTIMUM MTL. FOR ALL OTHER LAYERS.
     20 DO 200 I=1,N
        IF(T(I)-TL)21,30,30
     21 IF(T(I)-TC)30,30,23
C       IF NO MTL. IN THE ARRAY CAN BE USED FOR A NEXT LAYER, THE
C       OPTIMIZATION IS COMPLETED FOR J LESS THAN NL AND
C       THE J AND I LOOPS ARE EXITED TO STATEMENT 400
     23 MJ=I
        AKJ=AK(I)
        CJ=C(I)
        TJ=T(I)
        SUMI=SCT-AAK(J-1)*CC(J-1)*TJ+AKJ*CJ*TJ-AKJ*CJ*TC
        IF(SUMI-SUMCI)24,200,200
     24 SUMCI=SUMI
        M(J)=MJ
        AAK(J)=AKJ
        CC(J)=CJ
```

```
      TT(J)=TJ
      JJ=J
      ASU(J)=SU(I)
      ARO(J)=RO(I)
      SUM(J)=SUMT
      IF(SUMCI .EQ. SUMCJ) GO TO 400
   30 CONTINUE
  200 CONTINUE
  222 CONTINUE
      TL=TT(JJ)
      IF  (SUM(JJ)-SUMCJ)41,400,400
   41 SUMCJ=SUM(JJ)
      SUMCI=SUM(JJ)
      RET=SUM(J)+AAK(J)*CC(J)*TC
      NOOFW=NOOFW+1
  300 CONTINUE
C     END OF OPTIMIZATION
  400 CONTINUE
C     CALCULATION OF OPTIMUM VALUE (COST, DENSITY, OR THICKNESS)
      P=A*A*SUM(NOOFW)/0
C     SELECTION OF PRINT OUT OPTION FOR PROG=NXX=1,2,OR3
      GO TO (31,32,33),NXX
   31 WRITE (3,1007) NOOFW,P
      GO TO 34
C     NOTE---THE PROGRAM CALCULATES AND PRINTS OUT ALL OPTIMUM
C     CONFIUGRATIONS FROM 1 TO NL LAYERS.  HOWEVER, IF AN OPTIMUM
C     DOES NOT EXIST THE LAST NUMBER OF LAYERS CALCULATED, THIS
C     LAST CALCULATION WILL BE PRINTED OUT UNTIL NL PRINT OUTS HAVE BEEN MADE.
C     LAST CALCULATION WILL BE PRINTED OUT UNTIL NL PRINT OUTS HAVE BEEN
C     MADE.
   32 WRITE (3,1006) NOOFW,P
      GO TO 34
   33 WRITE (3,1008) NOOFW,P
   34 CONTINUE
      TT(1)= TH
      WRITE (3,1009)
      DO 500 J=1,NOOFW
      IF  (J-NOOFW)52,51,51
   51 DELT=TT(J)-TC
      GO TO 53
   52 DELT=TT(J)-TT(J+1)
   53 CONTINUE
C     CALCULATION OF LAYER THICKNESSES.
      X(J)=AAK(J)*A*DELT/ 0*12.0
      MJ=M(J)
C     PRINT OUT OPTIMUM LAYER DATA.
      WRITE(3,1003)M(J),X(J),TT(J),T(MJ),SU(J),ARO(J),AAK(J)
  500 CONTINUE
      IF(NPROG.GE.4.AND.NXX.LT.3) GOTO 61
   62 GO TO 63
   61 NXX=NXX+1
      GO TO 5
  888 WRITE(3,1004)
   63 CONTINUE
 7676 CONTINUE
      GO TO 8686
 7678 STOP
```

```
3030 FORMAT('1',48X,'SELECTION OF THE OPTIMUM'/52X,
    1'MULTI-LAYER'/48X,'COMPOSITE WALL CONFIGURATION'///)
1000 FORMAT (7I10)
1025 FORMAT('1',28X,'THE MATERIAL DATA IS AS FOLLOWS:'//12X,
    1'MATERIAL',8X,'MTL. TEMP.',10X,'CONDUCTIVITY',11X,'COST',
    212X,'DENSITY'/17X,'NO.',8X,'LIMIT DEG. F',7X,
    3'BTU/(HR-FT-F)      ',5X,'$/(FT-SQ)',7X,'LB./(CU.-FT.)'//)
1026 FORMAT(18X,I2,F20.1,F20.5,F20.3,F20.2)
1001 FORMAT (7F10.5)
1005 FORMAT (4F10.5)
1020 FORMAT(///28X,'THE GIVEN CONDITIONS ARE AS FOLLOWS:'/)
1021 FORMAT(32X,'SURFACE AREA IN SQ.-FT. (A)=',F9.1 /32X,
    1'HEAT TRANSFER RATE IN BTU/HR (Q)=',F9 .1/32X,
    2'HOT SURFACE TEMP. IN DEG. F (TH)=',F7 .1/32X,
    3'COLD SURFACE TEMP. IN DEG. F (TC)=',F7 .1/32X,
    4'NUMBER OF MATERIALS TO BE EVALUATED (N)=',I 2 /32X,
    5'MAXIMUM NUMBER OF LAYERS TO BE USED (NL)=',I2//28X,'FIND:')
1022 FORMAT(32X,'THE LOWEST COST WALL')
1023 FORMAT(32X,'THE LOWEST WEIGHT WALL')
1024 FORMAT(32X,'THE LEAST VOLUME WALL')
1007 FORMAT(///10X,'THE MINIMUM COST',1X,I2,1X,'LAYER WALL'/
    110X,'HAS A COST OF',1X,E10.4,1X,'DOLLARS'//)
1006 FORMAT(///10X,'THE MINIMUM WEIGHT',1X,I2,1X,'LAYER WALL'/
    110X,'HAS A TOTAL WEIGHT OF',1X,E10.4,1X,'POUNDS'//)
1008 FORMAT(///10X,'THE MINIMUM VOLUME',1X,I2,1X,'LAYER WALL'/
    110X,'HAS A TOTAL VOLUME OF',1X,E10.4,1X,'CUBIC FEET'//)
1009 FORMAT(2X,'MATERIAL',11X,'THICKNESS',8X,'HOT SURFACE',8X,
    1'MTL. TEMP.',11X,'COST',14X,'DENSITY',15X,'CONDUCTIVITY'/
    27X,'NO.',14X,'INCHES',8X,'TEMP. DEG. F',8X,'LIMIT DEG. F',
    39X,'$/(CU.-FT.)',7X,'LB./(CU.-FT.)',7X,'BTU/(HR-FT-F)')
1003 FORMAT(8X,I2,F20.5,2F20.1,F20.3,F20.2,F20.5)
1004 FORMAT(10X,'NO SOLUTION')
     END

/DATA
```

APPENDIX II

LISTING OF CYLINDRICAL WALL COMPUTER PROGRAM

```
/WAT4   MF120265,TIME=1,PAGES=10   D              MORATH, BILL                     JOB 685
        DIMENSION T(50),AK(50),SU(50),RO(50),C(50),M(50),ARO(50)
        DIMENSION TT(50),AAK(50),CC(50),SUM(50),X(50),ASU(50),PJ(50)
        REAL L
        READ (1,1000) N
C       N=NUMBER OF CARDS IN THE MATERIAL ARRAY (ONE CARD PER MATERIAL)
C       INPUT MATERIAL ARRAY
        READ(1,1005) (T(I),AK(I),SU(I),RO(I),I=1,N)
C       T(I)=MAXIMUM ALLOWED TEMP. OF MATERIAL (F)
C       AK(I)=CONDUCTIVITY OF MATERIAL (BTU/(HR-FT-F)
C       SU(I)=MTL.COST/VOLUME ($/(CU.-FT.)
C       RO(I)=MTL. DENSITY (LB./(CU.-FT.)
        READ (1,1000) NL,NINP,NPROG
C       NL=MAXIMUM NUMBER OF LAYERS TO BE ALLOWED IN THE FINAL RESULT
C       NINP=INPUT A 2 FOR PRINT OUT OF MTL. ARRAY.  INPUT A 1 FOR PRINT NO .
C       NPROG INPUT: 1,2,3,4 FOR MIN. COST MIN. WT., MIN. THICKNESS, ALL 3.
        WRITE(3,3030)
        IF(NINP.GE.2) GO TO 65
        GO TO 67
     65 WRITE (3,1025)
C       WRITE: MTL. ARRAY IF NPIN=1.
        WRITE(3,1026)(I,T(I),AK(I),SU(I),RO(I),I=1,N)
     67 CONTINUE
C       INPUT PROBLEM STATEMENT
   8686 READ(1,1001)TH,TC,Q,RIN,L,H
C       TH=CYLINDER TEMPERATURE (F)
C       TC=OUTSIDE FLUID TEMPERATURE (F)
C       Q=HEAT FLOW RATE THRU WALL (BTU/HR)
C       RIN=CYLINDER RADIUS
C       L=CYLINDER LENGTH IN (FT)
C       H=CONVECTION FILM COEFFICIENT  (BTU/(FT-SQ--F)
        WRITE(3,1020)
C       WRITE: PROBLEM STATEMENT
        WRITE(3,1021)Q,L,RIN,H,TH,TC
        WRITE(3,2021)N,NL
        GO TO (71,72,73,74 ),NPROG
     71 WRITE (3,1022)
        GO TO 2
     72 WRITE(3,1023)
        GO TO 2
     73 WRITE (3,1024)
        GO TO 2
     74 WRITE(3,1022)
        WRITE(3,1023)
        WRITE(3,1024)
      2 CONTINUE
        QH=6.28*L*RIN*H*(TH-TC)
        IF(QH.LE.Q) GO TO 997
        GO TO 996
    997 WRITE(3,1030) QH
        GO TO 8686
    996 CONTINUE
        NLIND=NL
        DO 7676 MMM=1,NLIND
        NL=MMM
C       SELECT OPTIMIZATION OPTION I.E.,NPROG=1,2,3,OR4
        IF(NPROG-4)4,3,3
```

```
      3 NXX=1
        GO TO 5
      4 NXX=NPROG
      5 CONTINUE
        GO TO (6,7,8),NXX
      6 DO 66 I=1,N
        C(I)=SU(I)
     66 CONTINUE
        GO TO 9
      7 DO 77 I=1,N
        C(I)=RO(I)
C       C(I)=DUMMY VARIABLE DEPENDING UPON NPROG=NXX=1,2,OR3
     77 CONTINUE
        GO TO 9
      8 DO 88 I=1,N
        C(I)=1.0
     88 CONTINUE
        GO TO 9
      9 CONTINUE
C       ESTABLISHING INITIAL VALUES FOR OPTIMIZATION:
        SUMCJ=10.0E6
C       START OF OPTIMIZATION - THE LOOP DO 300 J=1,NL,DETERMINES
C       IF N OR N+1 LAYERS ARE OPTIMUM.
        DO 300 J=1,NL
        IF(J-1)10,10,20
     10 CKM=10.0 E10
        XX=0.0
C       THE INNER LOOP DO 100 I=1,N, SELECTS THE MTL FOR THE HOT LAYER
        DO 100 I=1,N
        IF(T(I)-TH)13,11,11
     11 RSML=RIN
        CON=AK(I)
        DT=TH-TC
        RCR=CON/H
        XX=6.28*CON*L*DT/Q
        IF(XX .GT. 7.) GO TO 13
        RBIG=(EXP(XX)*RSML+RSML)/2.0
        IF(RCR-RBIG)43,44,44
     44 RBIG=2.0*RCR
     43 FB=Q*(1.*ALOG(RBIG/RSML)/CON+1./(H*RBIG))-6.28318*L*DT
        FBPR=Q*(1./(CON*RBIG)-1./(H*RBIG*RBIG))
        RBNEW=RBIG-FB/FBPR
         ITT=0
         IF(ABS(RBNEW-RBIG)/RBNEW .LE. 0.001) GO TO 45
         RBIG=RBNEW
        ITT=ITT +1
         IF(ITT .GE. 50) GO TO 999
        GO TO 43
     45 RBIG=RBNEW
        SUMI=C(I)*(RBIG*RBIG-RIN*RIN)
        IF(SUMI .GT. CKM) GO TO 100
        SUM(J)=SUMI
        CKM=SUM(J)
        RET=SUM(J)-C(I)*RBIG*RBIG
         RJ(J)=RBIG
        TT(J)=Q/(6.28*RBIG*L*H)+TC
        CC(J)=C(I)
```

```
      AAK(J)=AK(I)
      M(J)=I
      ASU(J)=SU(I)
      ARO(J)=RO(I)
      TL=T(I)
      II=I
      GO TO 100
   13 IF(I-N)100,144,144
C     STATEMENT 888 INDICATES THAT NO MTL. IN THE ARRY WILL
C     OPERATE AT THE HOT TEMP. OF THE PROBLEM STATEMENT.
  144 IF(XX .EQ. 0.0) GO TO 898
      IF(XX .GT. 7.) GO TO  777
  100 CONTINUE
      NODEW=1
      SUMCI=SUM(J)
      SUMCJ=SUM(J)
      GO TO 300
C     THE INNER LOOP DO 200 I= N, SELECTS THE OPTIMUM MTL. FOR ALL OTHER LAYERS.
   20 DO 200 I=1,N
      IF(T(I)-TL)21,30,30
   21 IF(T(I)-TC)30,30,23
C     IF NO MTL. IN THE ARRAY CAN BE USED FOR A NEXT LAYER, THE
C     OPTIMIZATION IS COMPLETED FOR J LESS THAN NL AND
C     THE J AND I LOOPS ARE EXITED TO STATEMENT 400
   23 IF(J .GT. 2)GO TO 49
      RSML=RIN
      RSMLX=RSML
      TS=TH
      GO TO 50
   49 RSML=RJ(J-2)
      RSMLX=RSML
      TS=TT(J-2)
   50 RSML=RSML*EXP(6.28*AAK(J-1)*L*(TS-T(I))/Q)
      IF(RSML .LT. RSMLX  ) GO TO 200
      CON=AK(I)
      DT=(T(I)-TC)
      RBIG=RJ(J-1)
   46 FB=Q*(1.*ALOG(RBIG/RSML)/CON+1./(H*RBIG))-6.28318*L*DT
      FBPR=Q*(1./(CON*RBIG)-1./(H*RBIG*RBIG))
      RBNEW=RBIG-FB/FBPR
       ITT=0
       IF(ABS(RBNEW-RBIG)/RBNEW .LE. 0.0005)  GO TO 98
      RBIG=RBNEW
      ITT=ITT +1
       IF(ITT .GE. 50) GO TO 999
      GO TO 46
   98 RBIG=RBNEW
      SUMI=RET+CC(J-1)*RSML*RSML+C(I)*(RBIG*RBIG-RSML*RSML)
      IF(SUMI .GE. SUMCI) GO TO 200
   24 SUMCI=SUMI
      JJ=J
      M(J)=I
      II=I
      AAK(J)=AK(I)
      CC(J)=C(I)
      ASU(J)=SU(I)
      ARO(J)=RO(I)
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      SUM(J)=SUMI
      TT(J-1)=T(I)
      RJ(J-1)=RSML
       RJ(J)=RBIG
      TT(J)=Q/(6.28*RBIG*L*H)+TC
      GO TO 200
   30 IF(I-N)200,400,400
  200 CONTINUE
      IF(SUM(JJ)-SUMCJ)41,400,400
   41 SUMCJ=SUM(JJ)
      TL=T(II)
      SUMCI=SUM(JJ)
      RFT=SUM(JJ)-CC(JJ)*RJ(JJ)*RJ(JJ)
      NOOFW=NOOFW+1
  300 CONTINUE
C     END OF OPTIMIZATION
  400 CONTINUE
C     CALCULATION OF OPTIMUM VALUE (COST, DENSITY, OR THICKNESS)
      P=3.14*L*SUM(NOOFW)
C     SELECTION OF PRINT OUT OPTION FOR PROG=NXX=1,2,OR3
      GO TO (31,32,33),NXX
   31 WRITE (3,1007) NOOFW,P
      GO TO 34
C     NOTE---THE PROGRAM CALCULATES AND PRINTS OUT ALL OPTIMUM
C     CONFIUGRATIONS FROM 1 TO NL LAYERS.  HOWEVER, IF AN OPTIMUM
C     DOES NOT EXIST THE LAST NUMBER OF LAYERS CALCULATED, THIS
C     LAST CALCULATION WILL BE PRINTED OUT UNTIL NL PRINT OUTS HAVE BEEN
C     MADE.
   32 WRITE (3,1006) NOOFW,P
      GO TO 34
   33 WRITE (3,1008) NOOFW,P
   34 CONTINUE
      WRITE (3,1009)
      DO 500 J=1,NOOFW
      WRITE(3,1003)M(J),RJ(J),TT(J),ASU(J),ARO(J ), AAK(J)
  500 CONTINUE
      IF(NPROG.GE.4.AND.NXX.LT.3) GOTO 61
   62 GO TO 63
   61 NXX=NXX+1
      GO TO 5
  888 WRITE(3,1004)
      GO TO 63
  777 WRITE(3,2222)
      GO TO 63
  999 WRITE(3,1031)
   63 CONTINUE
 7676 CONTINUE
      GO TO 8686
 7678 STOP
 1000 FORMAT (7I10)
 3030 FORMAT('1',48X,'SELECTION OF THE OPTIMUM'/52X,
     1'MULTI-LAYER'/48X,'COMPOSITE WALL CONFIGURATION'///)
 1025 FORMAT('1',28X,'THE MATERIAL DATA IS AS FOLLOWS:'//12X,
     1'MATERIAL',8X,'MTL. TEMP.',10X,'CONDUCTIVITY',11X,'COST',
     212X,'DENSITY'/17X,'NO.',8X,'LIMIT DEG. F',7X,
     3'BTU/(HR-FT-F)       ',5X,'$/(FT-SQ)',7X,'LB./(CU.-FT.)'//)
 1026 FORMAT(18X,I2,F20.1,F20.5,F20.3,F20.2)
```

```
 1001 FORMAT (7E10.5)
 1005 FORMAT (4F10.5)
 1020 FORMAT(///28X,'THE GIVEN CONDITIONS ARE AS FOLLOWS:'/)
 1021 FORMAT(32X,'HEAT TRANSFER RATE IN BTU/HR (Q)=',F9.1/32X,
     1'LENGTH OF CYLINDER IN FT. (L)=',F9.3 /32X,
     2'RADIUS OF CYLINDER IN FT. (RIN)=',F6.3 /32X,
     3'SURFACE CONDUCTANCE IN BTU/HR-FT. SQ.-F  (H)=',F6.3 /32X,
     4'SURFACE TEMPERATURE OF CYLINDER DEG. F (TH)=',F7.1 /32X,
     5'FLUID TEMPERATURE IN DEG. F (TC)=',F7.1)
 2021 FORMAT(32X,'NUMBER OF MATERIALS TO BE EVALUATED (N)=',I2,/32X,
     2'MAXIMUM NUMBER OF LAYERS TO BE USED (NL)=',I2//28X,'FIND:')
 1022 FORMAT(32X,'THE LOWEST COST WALL')
 1023 FORMAT(32X,'THE LOWEST WEIGHT WALL')
 1024 FORMAT(32X,'THE LEAST VOLUME WALL')
 1030 FORMAT('1','NO INSULATION IS REQUIRED.
     1  HEAT CONVECTION FROM THE CYLINDER IS',F6.1)
 1007 FORMAT(///10X,'THE MINIMUM COST',1X,I2,1X,'LAYER WALL'/
     110X,'HAS A COST OF',1X,F10.4,1X,'DOLLARS'//)
 1006 FORMAT(///10X,'THE MINIMUM WEIGHT',1X,I2,1X,'LAYER WALL'/
     110X,'HAS A TOTAL WEIGHT OF',1X,F10.4,1X,'POUNDS'//)
 1008 FORMAT(///10X,'THE MINIMUM RADIUS',1X,I2,1X,'LAYER WALL'/
     110X,'HAS A TOTAL VOLUME OF',1X,F10.4,1X,'CUBIC FEET'//)
 1009 FORMAT(2X,'MATERIAL',8X,'LARGE RADIUS',6X,'TEMP. AT LARGE',16X,
     1'COST',13X,'DENSITY',9X,'CONDUCTIVITY' /7X,'NO.',8X,
     2'OF LAYER FT.',6X,'RADIUS DEG. F',10X,'$/(CU.-FT.)',7X,
     3'LB./(CU.-FT.)',7X,'BTU/(HR-FT-F)' )
 1003 FORMAT(8X,I2,F20.5,F20.1,F20.3,F20.2,F20.5)
 1004 FORMAT(10X,'NO SOLUTION')
 2222 FORMAT(10X,'THE ALLOWED HEAT FLOW RATE IS TOO SMALL')
 1031 FORMAT(20X,'PROGRAM STOPED---50 ITERATIONS ON RADIUS')
      END
/DATA
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APPENDIX III

TABULATION OF CYLINDRICAL PROGRAM RESULTS

RESULTS OF COST OPTIMIZATION; CYLINDRICAL WALL

Q	cost ($)	R1 (ft)	R2 (ft)	R3 (ft)	R4 (ft)	T1 (°F)	T2 (°F)	T3 (°F)	T4 (°F)
		Single Layer Wall							
1000	87.33	2.820				111.7			
1150	63.42	2.460				114.9			
1500	37.06	1.988				124.0			
2000	22.39	1.668				138.2			
3000	12.04	1.400				168.3			
4000	8.08	1.282				199.4			
6000	4.75	1.174				262.8			
10000	2.47	1.094				391.1			
		Two Layer Wall							
1000	23.60	1.161	2.033			1900	115.7		
1150	18.43	1.138	1.851				119.8		
1500	12.02	1.104	1.600				129.9		
2000	7.91	1.077	1.419				144.9		
3000	4.53	1.051	1.259				175.9		
4000	3.24	1.038	1.186				207.5		
6000	1.99	1.025	1.116				271.2		
10000	1.08	1.015	1.064			1900	399.3		
		Three Layer Wall							
1000	19.52	1.161	1.830	1.998		1900	450	115.9	
1150	15.48	1.138	1.691	1.824				120.1	
1500	10.34	1.104	1.496	1.583				130.2	
2000	6.96	1.077	1.353	1.402				145.2	
3000	4.17	1.051	1.223	1.253				176.3	
4000	2.96	1.038	1.163	1.182		1900	450	207.8	
		Four Layer Wall							
1000	19.20	1.161	1.830	1.877	1.978	1900	450	353	116.1
1150	15.26	1.138	1.691	1.729	1.808				120.3
1500	10.22	1.104	1.496	1.521	1.577				130.4
2000	6.89	1.077	1.353	1.370	1.402				145.4
3000	4.14	1.051	1.223	1.234	1.250				176.5
4000	2.85	1.038	1.163	1.185	1.183	1900	450	353	207.7

Given Conditions: L=1, H=5, TC=100, TH=2200,

N=47, NL=4

MATERIALS :

Layer	Material	Q Range
1	31	all
2	21	all
3	6	all
4	4	all

RESULTS OF WEIGHT OPTIMIZATION, CYLINDRICAL WALL

Q	Wt. (lb.)	R1 (ft)	R2 (Ft)	T1 (°F)	T2 (°F)
		Single Layer Wall			
1000	79.73	2.711		111.7	
1150	58.37	2.376		115.4	
1500	34.52	1.936		124.7	
2000	21.05	1.636		138.9	
3000	11.42	1.382		169.2	
4000	7.68	1.270		200.3	
6000	4.53	1.167		263.8	
10000	2.47	1.094		391.1	
		Two Layer Wall			
1000	61.89	2.415	2.546	353	112.5
1150	46.45	2.152	2.252		116.3
1500	28.55	1.800	1.861		125.7
2000	17.99	1.554	1.591		140.0
3000	10.14	1.342	1.360	353	170.3

Given Conditions: L=1, H=5, TC=100, TH=2200,

N = 47, NL = 4

MATERIALS:

Layer	Material	Q Range
1	37	all
2	1	all (1000-3000)

RESULTS OF VOLUME OPTIMIZATION, CYLINDRICAL WALL

Q	Vol. (ft^3)	R1 (ft)	R2 (ft)	R3 (ft)	R4 (ft)	T1 (°F)	T2 (°F)	T3 (°F)	T4 (°F)
				Single Layer Wall					
1000	8.99	1.965				116.2			
1150	7.00	1.797				120.4			
1500	4.54	1.564				130.5			
2000	2.97	1.395				145.7			
3000	1.72	1.244				176.8			
4000	1.20	1.175				208.4			
6000	.73	1.110				272.2			
10000	.39	1.090				392.1			
				Two Layer Wall					
1000	7.79	1.763	1.866			450	117.1		
1150	6.14	1.637	1.719			↓	121.3		
1500	4.05	1.459	1.513			↓	131.6		
2000	2.69	1.328	1.362			↓	146.8		
3000	1.59	1.208	1.227			450	177.9		
4000	1.11	1.138	1.164			600	209.5		
6000	.69	1.090	1.104			600	273.1		
				Three Layer Wall					
1000	7.64	1.763	1.793	1.852		450	350	117.2	
1150	6.03	1.637	1.662	1.708		↓	↓	121.4	
1500	3.99	1.459	1.476	1.506		↓	↓	131.7	
2000	2.65	1.328	1.339	1.358		↓	↓	146.9	
3000	1.57	1.208	1.215	1.225		450	350	178.0	
4000	1.08	1.138	1.148	1.160		600	450	209.8	
				Four Layer Wall					
1000	7.62	1.763	1.793	1.839	1.851	450	350	170	117.2
1150	6.01	1.637	1.662	1.698	1.707	↓	↓	↓	121.4
1500	3.98	1.494	1.476	1.501	1.506	↓	↓	↓	131.7
2000	2.65	1.328	1.339	1.356	1.358	450	350	170	146.9

Given Conditions: L=1, H=5, TC=100, TH=2200,

N=47, NL=4

MATERIALS:

Layer	Material	Q Range
1	30	1000 to 10000
2	11	1000 to 3000
	17	4000 to 6000
3	15	1000 to 3000
	17	4000
4	42	1000 to 2000

VITA

The author, William Donald Morath, was born on January 13, 1929, in St. Louis, Missouri. He received his primary and high school education in University City, Missouri, and received a Bachelor of Science Degree in Mechanical Engineering from Purdue University in 1955. Since graduation he has worked as a Mechanical Engineer at Rocketdyne Corp., Marquardt Corp., Vickers Inc., and McDonnell Douglas Corp. He is presently employed as an assistant professor in the Engineering and Mathematics Department of Meramec Community College in Kirkwood, Missouri. He enrolled at the University of Missouri-Rolla, St. Louis Graduate Engineering Center in the fall of 1965, and attended the University of Missouri-Rolla in the summer of 1968.